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Design and Analysis of Purely Functional Programs. Second Edition
Christian Rinderknecht

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Purely Functional Programs
Second Edition

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