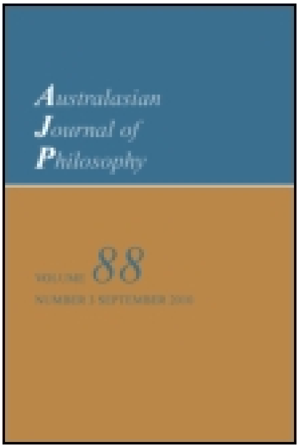

Australasian Journal of Philosophy
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/rajp20

Review of 'Inconsistent Geometry', by Chris Mortensen

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Version of record first published: 02 Jul 2012.

To cite this article: Zach Weber (2012): Review of 'Inconsistent Geometry', by Chris Mortensen, Australasian Journal of Philosophy, 90:3, 611-614

To link to this article: http://dx.doi.org/10.1080/00048402.2012.695383

outstrips both the idea that one seems nicer than the other, as well as all the logical and causal facts.

I'm pretty sceptical about what strikes me as a reification of the pragmatic side of explanation, and maybe Daly hasn't devoted as much space to that scepticism as I might have, but that's not something to pursue in the context of this review. Rather, it's that there is a methodological question that this raises which is much neglected. How can you argue for one view rather than the other? You can't use the principles of explanatory virtue, because that is precisely what is at issue. Perhaps you can use other non-deductive argument forms of the kind discussed. But how do we evaluate them? Should philosophical methodology be treated as a kind of package, where we should look for the most coherent whole? What if some methodologies are neither supported nor undermined by others? There is a general question of meta-methodology that really deserves consideration.

In any case, this book is very welcome. There's nothing quite like it and, for its scope and its clear and balanced approach, I think it's something that every advanced undergraduate would benefit from reading, and many other philosophers as well. I'll certainly be trying it out as the basis for a course.

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Mortensen, Chris, *Inconsistent Geometry*, Milton Keynes: College Publications, 2010, pp. 174, US$35.50 (paper). http://dx.doi.org/10.1080/00048402.2012.695383

A startling but rarely mentioned theorem of classical model theory states that *every inconsistent theory is categorical*. What a coup for the inconsistent mathematician! But no. 'All' models of an inconsistent theory are isomorphic, only because classically there *are no* such models. Classical theory teaches, in a slogan, that the inconsistent has no structure.

In his 1995 book, *Inconsistent Mathematics*, Chris Mortensen set out to refute that thesis, by embedding various branches of mathematics in paraconsistent logics and demonstrating that they can exhibit inconsistent but non-trivial structure. In *Inconsistent Geometry*, a stand-alone sequel, he takes on the challenge much more directly, by displaying a whole lot of structures—beautifully rendered 'impossible pictures'—alongside a mathematical apparatus that allows these geometric objects to be interestingly inconsistent.

Mortensen's book is a significant accomplishment on several counts. First, it is a highly original contribution to the fledgling field of inconsistent/paraconsistent mathematics. Second, it is an investigation of inconsistency in a surprising area; after all, say what you will about truth predicates or naïve set theory, but *geometry*—that paragon of clear and distinct ideas as a site for contradiction? Third, the book is an uncompromising and technically demanding work that could have easily been pitched as light airplane reading, in the vein of pop-philosophical musings on fractals. Instead it is a serious, confident and unapologetic exposition of new mathematics of the inconsistent.

It should be noted straightaway that, while singly authored, the book has two other contributors. Mortensen's collaborators at The University of Adelaide, Steven Leishman and Peter Quigley, co-wrote one chapter each, and Leishman drew almost all the pictures in the book. This is no small contribution, as *displaying* paradoxical images is one of the central attractions and arguments of the piece. My favourite is 'Superstructure in perspective' [154].

Inconsistent Geometry is in two parts. The first half is about how geometric aspects of topology and algebra can be related to logic, by reading logics off mathematical objects. For example, the very natural paraconsistent logic, P3, turns out to be a *closed set logic* (akin to intuitionistic open set logic). It is shown how standard topological separation axioms can be used to control conditions on identity, =; one arrangement gives us a 'ring of fire' model, in which any set S that is closed but not open is surrounded by a boundary of non-self-identical elements $a \neq a$. Chapter 3 shows how Łukasewicz logics can be given a semantics with group theory, and Chapter 4 (co-authored with Quigley) extends the thought by taking a brief detour into games.

A trio of chapters develops algebraic geometry. In the chapter on 'Symmetries', the idea of inconsistency is motivated by considering rotations. Take a square in the plane; now rotate it 90 degrees. Very roughly, the idea seems to be that the resulting square is now *different*, because it has been rotated, but it is also in some obvious sense *the same*. We then get into the formalism. Using a result known as the *collapsing lemma*, it is possible to 'inconsistentize' a consistent theory, by taking some objects that are non-identical, and identifying them by an equivalence relation. The result is an inconsistent but non-trivial theory.

As a bit of reassurance, Mortensen shows that the consistent theory can always be recovered by using the Routley Star, *, defined as such: where S is a set of sentences, $S^* = \{p\text{: the negation of p is not in S}\}$. Given double negation elimination, the effect of the Routley Star on a set of sentences is to remove inconsistent pairs $\{A, \neg A\}$. Factor groups can be described in terms of the Routley Star, which itself starts to look very much like an object familiar from homology theory (for those who find anything familiar in homology theory)—the boundary homomorphism, satisfying the fundamental theorem that *the boundary of a boundary is zero* [64].

Most of the theorems in this first part of the book are 'about' or 'over' various mathematical theories, rather than being results *of* those theories. So they are mostly logical results in model theory, not new theorems of geometry or algebra *per se*. Mortensen distinguishes between *mathematical* triviality and *logical* triviality. The latter is when a theory contains every sentence, and is absolutely unacceptable. The former is when a theory contains every sentence of the form $x = y$, and is more subtle. Similarly, and as in the book's predecessor, Mortensen is always concerned to check conditions under which a theory obeys substitution of identicals. Again, then, this part of the book is a logician's geometry.

The second half of the book is where it really comes alive. This is in no small part due to the profusion of mind-bending impossible pictures adorning most of the pages. Here 'impossible' means *real* pictures that exhibit some kind of logical inconsistency, and Mortensen and Leishman have produced a vast array.

The most technically mature and compelling material is in Chapters 9–12, which subject the well-known Necker Cube to an exhaustive analysis using linear algebra. Here we start to glimpse the prospects for full-scale inconsistent geometry: for with the machinery of matrices and determinants comes the language of vectors and similar higher-level geometric concepts. Mortensen is able to characterize the inconsistency of a Necker Cube—and its wild generalization, the Necker *Chain*—in terms of solutions to equations. Two sorts of inconsistency are identified, *local* and *global*, and then the concepts are linked in a satisfying theorem (that inconsistency and incompleteness can be characterized entirely by the existence of solutions to an equation [95]). Methods are presented for calculating the degree of inconsistency of a Necker, and some more uses for the Routley Star are found (though on the latter, I am less impressed than Mortensen is about the ubiquity of the Routley functor).

Final chapters on 'The Triangle', 'The Stairs', and 'The Fork' go some way more towards showing that these objects can be understood using inconsistent methods. Each chapter considers the problem in three phases: logic, cognition, and mathematics, providing a suggested methodological blueprint for later work. The logical phase aims at describing the picture in question so as to make some contradiction explicit. The cognitive phase comments on how we tend to perceive such pictures, with appeals to several studies in experimental psychology by Cowan and Pringle, carried out in the 1970s. And then the mathematics is meant to extract formal properties.

The main *philosophical* claim of the book, implicit in the method just described, is that our experience of seeing impossible pictures is best described with an inconsistent mathematical theory. Looking at, for instance, a Schuster Fork, the phenomenology seems to involve paradox in some way; therefore, these are best met with paraconsistency. Mortensen wants to align himself with so-called 'weak' paraconsistency, avoiding full-bore dialetheism, by making the target cognitive: 'The content of human geometrical experiences and reasoning [is] inconsistent' [4]. Ergo the right mathematics to describe these experiences is inconsistent—or so the claim goes.

But that conclusion does not obviously follow. For example, trees are made out of wood, but there is no reason to demand that a *theory* of trees be made out of wood. Why think that the intuitively paradoxical nature of impossible pictures should be represented as a logically inconsistent set of sentences? Does formal inconsistency really preserve the target? The answer comes, not in the purported terms of cognition, but in terms of *truth*: 'Impossible images must be shown to be inconsistent, by providing enough logical analysis to display a proposition A and its negation $\neg A$ as part of the content of the image' [69]. The content of the image may well include something cognitive, but that is not enough. Upon careful exposition, Mortensen seems to accept that *mathematical* facts about (and not merely our cognition of) some pictures are truly inconsistent [5]. Whatever perceptual content comes to (there is a somewhat obscure analogy to perspective and projective geometry made at [74]), it seems to me that some appeal to truth is needed to warrant paraconsistent analysis of these pictures. The cognitive phenomenon may be important, even necessary for motivating inconsistent geometry, but it does not appear to be sufficient. In the end, truth is required.

The main *mathematical* claim of the book is that all impossible pictures fall under one of four categories. This is a step back from Mortensen's original hypothesis, that all impossible pictures are *occlusion illusions*: pictures that can be rendered consistent by reversing at least one order in which objects appear in front of each other. Not all impossible pictures are occlusions, though, leaving the project with only a broad, ostensive classification, and not a deeper or more general definition. For what it is worth, it strikes me that Mortensen comes tantalizingly close to identifying a unifying property between the four broad types: might they all be *non-orientable* surfaces—all containing somewhere a Möbius Band, or 'twist', that delivers the impossibility? Chapter 14, on 'The Stairs', appeals to the idea of a twist as generating the object; but the analysis stops there. It would be highly desirable and interesting, at any rate, to extend the ideas in this book towards a more general theory; what we have for now, at least, are field notes on a new science. For those whose interests are piqued, clearly most of the work in inconsistent geometry lies in exciting days ahead.

It should not sound the least bit unkind to say that working in this area is like skating at the edge of reason—a phrase once used by Hans Segan to describe his own work in the mathematics of space-filling curves. I am happy to report that Mortensen's rigorous and measured work stays on the right side of madness, though after reading his book, I can't say for certain which side the right side is.

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