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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Clinton Conley (Managing Editor), Mark van Atten, Benno van den Berg, Thomas Colcombet, Samuel Coskey, Bradd Hart, Bernard Linsky, Antonio Montalbán, Rahim Moosa, Christian Retoré, and Nam Trang. Authors and publishers are requested to send, for review, copies of books to *ASL, Department of Mathematics, University of Connecticut, 341 Mansfield Road, U-1009, Storrs, CT 06269-1009, USA*.

N. FRANCEZ, ***Proof-theoretic semantics***, Studies in Logic, vol. 57, College Publications, London, 2015, xx + 415 pp.

This book, addressed to an audience of mathematicians, computer scientists, philosophers, and linguists, presents in a comprehensive way the viewpoint of proof theoretic semantics. According to proof theoretic semantics,[1] the meaning of a sentence lies in its proof-conditions rather than in its truth-conditions. The first part of the book is devoted to the development of this idea for logical formulas, constituting an impressive and timely work of synthesis. The second part focuses instead on proof-conditions for sentences belonging to fragments of natural language and—to our knowledge—represents one of the few attempts to specify the meaning of natural language sentences in a proof-theoretic way. The mainstream formal semantics for natural language is in fact model-theoretical, so that Francez's contribution is more than welcome. This review will follow the structure of the book.

§1. First part of the book. The first and the second chapters present a succinct and convincing account of the motivations behind the idea that meaning should be explained in terms of proof-conditions. They also offer an introduction to various systems of natural deduction. Systems of natural deduction are the main formal tools used to characterise the semantics of a sentence throughout the book; the semantics of a sentence A is defined in terms of *canonical derivations* of the sentences. Canonical derivations of A are—roughly speaking—detour-free derivations of A having a particular structure, e.g., whose last rule is an introduction rule for the main connective of A. Various systems of natural deduction are introduced together with their meta-theoretical properties. However, proofs of these properties are omitted. In our opinion the exposition would benefit from their inclusion. Some of those proofs are classic, and punctual references are given; nonetheless, both experienced logicians and neophytes would benefit from further exposition.

The third chapter exposes some criticisms that are raised against the idea that the meaning of the logical constants lies in their deduction rules. Francez illuminates how these criticisms are overcome by the partisans of proof theoretic semantics. In particular we find a detailed discussion of the *tonk* connective introduced by N. Prior.[2] This connective is defined in terms of deduction rules and has the unwelcome property of trivialising the deductive system. Francez exposes how we could prevent the definition of this kind of connective by imposing

[1]Some classical references are D. PRAWITZ, ***Towards A Foundation of A General Proof Theory***, Studies in Logic and the Foundations of Mathematics, vol. 74 (1973), pp. 225–250 and M. A. E. DUMMETT, ***The Logical Basis of Metaphysics***, The William James lectures, Harvard University Press, 1993.

[2]N. PRIOR, *The runabout inference-ticket*. ***Analysis***, vol. 21, no. 2, pp. 38–39.

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certain conditions on the rules that are regarded as meaning-conferring. In particular the chapter contains a detailed discussion of the concept of *harmony* between introduction and elimination rules and a comprehensive comparison between the various definitions of harmony that one can find in the proof theoretic semantics literature.

The fourth chapter exposes some—rather exotic—material even for a partisan of proof-theoretic semantics. Proof-theoretic semantics has its roots in constructive mathematics and the aim of the material presented in this chapter is to give a proof-theoretical semantics of classical logic. Francez presents two main alternatives: systems of multi-conclusion natural deduction that remind one of sequent calculi systems and natural deduction enriched by a primitive operation of denial *distinguished from negation*. This chapter contains also a detailed discussion of the role of *structural rules* in the proof-theoretic semantics enterprise. This kind of discussion is quite uncommon in the proof-theoretic semantics literature and deserves appreciation. In our opinion this is one of the chapters that would have benefited from the inclusion of proofs concerning meta-theoretical properties of the system presented. The book insists meaning lies in proofs so, to better understand these quite exotic systems, proofs of their properties would be more than welcome.

In chapter five, which is called proof semantic values, N. Francez' personal viewpoint starts to emerge more conspicuously. Usually the partisans of proof-theoretic semantics states that the meaning of the logical constants are given *implicitly* by their canonical proof conditions and the focus of people working in proof-theoretic semantics has been on finding a suitable definition of logical consequence cast in terms of canonical proof conditions. The author however does the exact opposite. The semantic values of formulas are defined explicitly in terms of set of canonical proofs constructed in some formal system of natural deduction and the notion of logical consequence is not studied in detail. This chapter is surely one of the most interesting in the book, though unfortunately it is also one of the hardest to follow. The notation is quite heavy and some conceptual clarification of the definitions would be welcome. The sixth chapter details some alternative views on proof-theoretic semantics that use sequent calculus rather than natural deduction as the meaning-conferring proof system. An interesting view heavily inspired by logic programming is also presented.

§2. Second part of the book. The second part of the book is concerned with developing Proof Theoretic Semantics for small fragments of natural language. After a short introduction in chapter seven, chapter eight defines natural deduction proof systems for fragments of natural language containing sentences with a simple structure[3] e.g., subject, verb object, quantifiers and relative clauses. In particular the author gives natural deduction systems for natural language generalised quantifiers like "all" and "some." Generalised quantifiers are usually defined in terms of relations between subsets of an arbitrary universe U. For instance $U \models All(A, B)$ iff $[|A|] \subseteq [|B|]$ in U. Francez instead designs natural deduction rules for the two quantifiers. He also states that one can prove the normalisation theorem for a natural deduction system including these rules. Again an explicit proofs of this claim would have been warmly welcomed.

Chapter nine is an interesting approach to the meaning of subsentential units and could be read as an attempt to formalize Frege's *context principle*. Concretely the author shows how to specify the meaning of subsentetial expressions up to words from the meaning of the sentences in which they occur by using a syntactic analysis of the sentence carried out in type theoretical grammar.

Chapter ten presents a detailed study of the proof-theoretic semantics of determiners encompassing generalised quantifiers. Francez takes a particularly interesting approach to the phenomena of monotonicity, one which does not rely on an external—to the proof system—partial ordering on the expressions. We think that a comparison of this approach to the one of natural logic would be fruitful.

Chapter eleven is devoted to the semantics of intensional transitive verbs. These are verbs like *seeks*, which occur in sentences like *John seeks a killer*. Such sentences are semantically

[3] Like it is common in natural logic see L. Moss, ***Natural Logic***, second ed., Chapter 18, John Wiley and Sons, 2015, pp. 561–592.

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ambiguous between a specific reading and a general reading. The treatment of intensional transitive verbs is notoriously difficult in model theoretic semantics and the solution proposed by Francez is elegant and intuitive.

Chapter twelve explains how one can handle a natural language sentence's *implicit* context of utterance in proof-theoretical terms. The truth conditions of a natural language sentences usually depend upon a certain context of utterance. This context restricts the quantificational scope of the generalised quantifier to a certain given situation. For instance the sentence "all the bottles are empty" usually means something like "all the bottles in the fridge are empty." First Francez explain why it is hard to deal with context dependence in model theoretic terms. Then he propose to model context as assumptions $H_1, \dots, H_n$ in a natural deduction derivation. The context dependence of a sentence A is modelled by restricting the introduction rule of the main symbol of the first order logic formula representing A to the context $\Gamma, H_1 \dots H_n$ where Γ is arbitrary. This chapter also contains natural deduction rules for the ι operator of Russel, which is typically used to express denotational uniqueness e.g., used to capture the logical form of a noun phrase like "The president of France." The book ends with a brief afterword summarising the main proposal while pointing the way to open problems in proof-theoretic semantics.

To sum up, this book offers a good introduction to proof-theoretic semantics. An introduction which has the virtue of not lingering on endless philosophical discussions and also the remarkable virtue of being the only comprehensive introduction on the topic. We would have preferred this book to be more self contained, e.g., by detailing the proof of the propositions but the synthesis provided by Francez and the biography on which he relies are quite impressive. We have particularly appreciated the second part of the book: it is a breath of fresh air in formal semantics, an area which is heavily "dominated" by the model-theoretic approach to meaning.

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Francez, Nissim (IL-TECH-C)

★**Proof-theoretic semantics.**

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College Publications, [London], 2015. *xx*+415 *pp. ISBN* 978-1-84890-183-4

This book is intended for researchers and Ph.D. and master's students in mathematical logic (proof theory), in computer science (type theory, functional programming), in linguistics (formal semantics), and in philosophy of language (logic and language). It develops the interesting viewpoint according to which the meaning of the logical operations (implication, conjunction, disjunction, existential and universal quantification) lies not in their model-theoretic interpretations but rather in the deduction rules. Accordingly, the meaning of compound formulas is viewed as their proofs from simpler formulas. In the second part, the book treats on a par the meaning of natural language sentences, phrases, and words. For instance the meaning of grammatical words like determiners is viewed as the way they affect proofs of sentences or formulas. So this book is focused on the compositional aspects of meaning, presented in a top-down approach. The meaning of atomic formulas is not discussed—and accordingly the "root" meaning (lexical semantics) of common nouns, adjectives, verbs, and the relationship between them are also left out. Despite some heavy notation here and there, or some missing comparisons (Prawitz grounds, natural logic), this book is worth reading, as it presents an interesting and original research direction.

Chapter 1: Introduction. An opinion-minded introduction of nearly 50 pages presents the subject, in particular the advantages of proof-theoretic semantics compared to the more standard model-theoretic semantics. This introduction gives the general principles of proof-theoretic semantics (as well as an outline of natural deduction), and it is a convincing defense of proof-theoretic semantics.

Part I: Proof-theoretic semantics in logic.

The first part of the book is mainly on natural deduction, as a deductive system which captures the meaning of connectives. This view has initially been explored by M. A. E. Dummett and D. Prawitz (and can be said to have been launched by G. Gentzen). I regret that the recent works of Prawitz on *grounds*, which is listed in the bibliography, is not discussed.

Chapter 2: A variety of logics. Of course, the presentation starts with intuitionistic logic, which is *the* natural logic when working with natural deduction. The chapter thereafter addresses natural deduction systems for other logics: classical logic, relevant logic, modal logic, etc. Those later natural deduction systems do not enjoy as many properties as the intuitionistic system does. For instance, classical logic needs *reductio ad absurdum*, preventing the sub-formula property to hold anymore.

Chapter 3: The basis of acceptability of *ND*-rules as meaning-conferring. This chapter presents objections to deduction rules as meaning-conferring, in particular the objection raised by the *tonk* connective due to A. N. Prior: this connective is defined by two (strange) rules, but from those rules it is impossible to grasp what tonk may mean. Thereafter the author presents important proof-theoretic notions, namely harmony and stability, which mainly concern the symmetry between introduction and elimination rules. The search for such principles, initiated by Prawitz, leads to systems which avoid problems like the ones raised by Prior's tonk.

Chapter 4: Alternative presentations and the justification of classical logic. This chapter presents variants of natural deduction for classical logic, in order to have meaning-conferring rules for classical connectives. I personally agree with the book *Proofs and types* [Cambridge Tracts Theoret. Comput. Sci., 7, Cambridge Univ. Press, Cambridge, 1989; MR1003608] by J.-Y. Girard, P. Taylor and Y. Lafont that the symmetries of classical logic, namely the De Morgan principles, are better expressed in sequent calculus. Although those systems better match the principles defined in the previous chapter, I personally find them far-fetched, and I have doubts that normalization (cut-elimination) and the sub-formula property can easily be established for such complicated systems.

Chapter 5: Proof-theoretic semantic values. This chapter maps formulas to specific sets of proofs, canonical proofs of various kinds that favor introduction or elimination rules. The author should have better explained what semantic value is assigned to atomic formulas which cannot be decomposed: what are such proofs, and what are their hypotheses? Thereafter the lexical meaning of the logical symbols is given in proof-theoretic terms. It is said to be in the Montague style, but R. Montague never gave much importance to formulas, and even less to proofs—the view of formulas as well-formed proofs (or as typed λ-terms) is due to A. Church (1940) rather than to Montague (1970s).

Chapter 6: Other developments. Here the author presents some extensions and adaptations of the proof-theoretic view developed in the previous chapter to innovative and/or exotic proof systems (with complicated notation).

Part II: Proof-theoretic semantics for natural language.

Chapter 7: Introduction II. This chapter introduces the second part by giving the principles of proof-theoretic semantics for natural language. This second part is shorter than the first and is devoted to the proof-theoretic semantics of natural language sentences and of sub-sentential expressions (words and constituents). I regret that the connection with natural logic is not discussed.

Chapter 8: Proof-theoretic sentential meanings. This chapter defines the semantics of a natural language sentence matching a simple pattern (subject-verb-object, first with quantifiers, and next with relative clauses) as a proof of this sentence from simpler sentences. A particularity due to S. Read is to use a specific predicate(s) for the copula. I think this approach could be connected to natural logic.

Chapter 9: Proof-theoretic meanings of sub-sentential phrases. This chapter presents an interesting viewpoint which could be called "reverse Montague semantics": indeed, it shows how one can assign meaning to a sub-sentential expression down to its elementary units (words) from the meaning of the whole sentence and its syntactic analysis (a proof in a type logical grammar). This top-down approach could be compared to discourse representation theory (DRT), but a DRT going up to words, thus defining the logical contribution of words.

Chapter 10: A study of determiners' meanings. This chapter presents an interesting modeling of determiners which encompasses usual quantifiers, definite articles, etc. Somehow this is the proof-theoretic pendant of *generalized quantifiers* which have been thoroughly studied in model-theoretic semantics. Numbers are evoked ("*at least* n*, at most* n*, exactly* n"), but they would deserve a more thorough study. Plurals and mass nouns could also be addressed. Here as well the book could include a comparison with natural logic, in particular with the work of Larry Moss.

Chapter 11: Opacity: intensional transitive verbs. This chapter makes use of specific variants of predicates to model opaque contexts, for which the existence of some entity is doubtful ("seeking a secretary / a unicorn"). Here as well the book develops a rather simple solution in proof-theoretic semantics for a question which is quite difficult in the model-theoretic approach.

Chapter 12: Contextual meaning variation. In this chapter the author studies how the proof-theoretic framework can handle contexts, which can be viewed as assumptions, and which unfortunately are often implicit. I appreciated that the treatment is rather original, using subnectors and in particular the Peano-Russell *iota* to restrain the variation of variables, and to select elements from a context.

A brief conclusion (afterword) sums up what has been proposed in proof-theoretic semantics by the author and his view of what is left to be done.

Although parts of this book may seem to address rather obscure or far-fetched deductive systems, on the whole *Proof-theoretic semantics* by Nissim Francez presents a highly interesting and underrepresented viewpoint on semantics, be it for logical formulas or for natural language sentences, which are treated on a par (this parallel as well is original and interesting). The reviewer would have appreciated some additional comparison with existing work (grounds, natural logic, lexical semantics).

Christian Retore

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Nissim Francez: *Proof-theoretic Semantics*
College Publications, London, 2015, xx+415 pages

During the second half of the twentieth century, most of logic bifurcated into model theory and proof theory. Model theory, as established by Tarski & Co., was considered as a matter of "semantics": it investigated the relationship between formal languages and the domains of entities about which the languages were supposed to be. Proof theory, on the other hand, was taken to be a matter of "syntax": not concerning what the formulas of formal languages are about, but about certain relations among them. Hence, when there appeared the term *proof-theoretic semantics* (PTS), it sounded quite paradoxical: how could there be a "syntactic semantics"?

The solution to this quandary lies, I am convinced, in the elucidation of the misleading role the term "syntax" has played within modern logic (and philosophy). Primarily, syntax is a theory of "well-formedness"—of the delimitation of the range of expressions which make up a given language. In this sense, syntax indeed has nothing to do with semantics and it would be futile to try to base a semantics on it. However, the term "syntax" has also been used to refer to inferences, derivations and proofs, and if considered in *this* sense, it is no longer so clear that it is unrelated to semantics. Indeed, the second half of the twentieth century also witnessed the rise of the so-called use-theories of meaning, at least some of which identified meaning of an expression specifically with the role conferred on it by the inferential rules governing its proper employment.[1]

Proof-theoretic semantics is also closely connected with the search for the proper semantics of intuitionistic logic. While classical logic has the natural truth-functional semantics, there was, for some time, no such canonical semantics for intuitionist logic. What was subsequently to become accepted as its adequate semantic account was its so-called BHK-interpretation (see Troelstra & van Dalen 1988): the idea that the semantics is based on the concept of proof. This idea is usually incorporated into PTS in such a way that the meaning of a sentence is considered as the set of all its proofs (or all its "canonical" proofs) and that logical connectives express ways to combine proofs of components into proofs of a compound. The term "proof-theoretic semantics" was introduced by Schroeder-Heister

[1] See Peregrin (2006a) for a discussion.

(1991) and its development is often associated with Prawitz (2006). Francez's book presents its elaboration not only for the formal languages of logic, but also for natural languages.

PTS built on this basis seems to pose two problems. Firstly, the association of the meaning of a statement with the set of its proofs appears to be epistemologically unrealistic: do we want to say that whoever understands a sentence is bound to know all the ways to prove it? And secondly, even if we accept this, the theory gives us meanings of sentences, but what about those of sub-sentential expressions?

Compare the situation with the well-known origin of model-theoretic semantics (MTS) for the languages of logic and its extension to natural language initiated by Montague (1974). Here, the starting point is the Frege's (1891) idea concerning explicating meanings of predicative expressions as functions from objects to truth values, which led to the standard truth-functional semantics for logical operators. This then led on to the general idea that the meanings of expressions of all other categories, save sentences and names, are functions built on the basis of the denotations of sentences (truth values) and names (objects). (For a very general formal language this was proposed, for the first time, by Church 1940.) Montague then provided his elegant semantic treatment of a fragment of English, which secured MTS a place on a philosophical pedestal.

But note an important feature of the Montagovian MTS: it was not extensional because the denotations of sentences were not their truth values; they were rather functions from possible worlds to truth values (or the sets of possible worlds that the functions characterize). Despite this, logical connectives could, in effect, retain their truth-functional denotations. Thus, even if we were to explicate the denotations of sentences as the sets of their proofs, it need not follow that if PTS were to follow in Montague's footsteps, the denotations of logical connectives would have to be something as monstrous as functions from pairs of sets of proofs to sets of proofs. And indeed it is the BHK-interpretation that indicates to what such denotations could be reduced: to methods of combining proofs that could be quite simple.

In view of this, we can put, and this is something Francez shows very explicitly in the book, the whole Frege-Church-Montague functional machinery into the services of PTS. On the level of propositional logic, we have just to assume that we have denotations of statements (the sets of their proofs) and derive the denotations of sentential operators as corresponding functions. Then we can try to reduce them to something simpler; in the case of conjunction it could, for example, be combining proofs of the two conjuncts into the proof of the conjunction, which could be nothing more complicated than putting the two proofs beside each other. Thus, the meaning of "∧", for example, may be the function which maps two sets of proofs, P_1 and P_2, on the set of proofs that contains, for every proof D_1 of a formula A_1 that belongs to P_1 and every proof D_2 of a formula A_2 that belongs to P_2, the proof

$$\frac{D_1 \qquad D_2}{A_1 \wedge A_2}$$

The situation is a little more complicated on the level of predicate logic, where Church made use of one more category the denotations of which were primitive, namely names (which, according to him, denote individuals—elements of a universe). Here Francez's approach diverges from the model-theoretic one: the category of expressions he chooses as the other primitive one are (individual) variables. A variable, according to him, denotes itself. As a result, a quantifier takes the denotation of a sentence (the set of its proofs) plus that of a variable (the variable itself) to the denotation of a quantified sentence. This, of course, presupposes that the category of sentences includes open formulas. In this way, the meaning of "∀", for example, is a function which maps a set of proofs P and a variable *v* on the set of proofs that contains, for every proof D of a formula A that belongs to P (and such that *v* is not in any premise or undischarged assumption of D), the proof

$$\frac{D}{\forall v A}$$

It turns out, however, that this version of PTS has a property that may be seen as problematic: the meanings of sentences, *viz.* the sets of their canonical proofs, turn out to be overly fine-grained. (For example, the meaning of $A_1 \wedge A_2$ comes out as different from that of $A_2 \wedge A_1$. This may make some sense for a natural language, but much less for a logical language in which the two sentences are provably equivalent and provably intersubstitutive w.r.t. logical equivalence.) Hence it would seem that what would fare better in this respect would be the identification of the meaning of a sentence with the set of *grounds* of the sentence: sets of all sets of formulas from which the formula is derivable. (Also we might think about including only *maximal* grounds, which would then be not so far from possible worlds, and PTS would come slightly closer to MTS.) It is a pity that Francez does not elaborate on this idea.

Francez uses the concept of ground also for the definition of proof-theoretical consequence: *A* is a consequence of *X* iff everything that is a ground for *X* is a ground for *A*. Again, it seems to me to be a pity that Francez does not tell us more about the proof-theoretical relation of consequence defined in this way. (Usually it is noted that there is a gap between *derivability*, as a proof-theoretical matter, and *consequence*, which must be defined model-theoretically (see, e.g., Etchemendy 1990). Carnap (1934) tried to account for this gap in purely proof-theoretic terms (see Peregrin 2014, Ch. 7); and it would be nice to learn what ambitions Francez has using his definition.)

The second part of Francez's book applies PTS to natural language, thus creating an antipode to the Montagovian MTS. Some of the ideas already embodied into PTS for the languages of logic can be straightforwardly transferred to natural languages, but in some respects natural languages are different. In particular, we can treat at least some of the connectives on a par with their logical counterparts; but the way quantification operates in natural language is very different from the standard Fregean quantifiers embraced by logic.

Francez, nevertheless, approaches the situation analogously to that of the formal languages. He enriches the fragment of natural language by using "individual parameters", which play a role somewhat analogous to that played by variables in formal languages. (Thus the whole language he works with is comprised of natural language plus "open" sentences that can be assembled from elements of natural language and parameters.) And though the mechanism of quantification is different, Francez's way of coping with it proof-theoretically is quite similar: the proof of a general statement builds on the proof of the corresponding statement with an indeterminate individual parameter.

One of the crucial features of Montagovian formal semantics was that it accounted for intensional contexts, that by engaging possible worlds it surpassed the limits of extensional semantics (see Peregrin 2006b). The proof-theoretic account of Francez has no lesser ambitions: it, too, aspires to account for the intensionality of natural language. However, here the method differs greatly from the model-theoretic one. What does the work here is nothing like possible worlds. Francez introduces a new kind of individual parameters, which he calls *notional parameters*. These parameters have inferential properties different from ordinary individual parameters. For example, while *John finds a unicorn* is introduced on the basis of *John finds x* and *x is a unicorn* (hence it follows that there is something that is a unicorn), the grounds of the introduction of *John seeks a unicorn* are different: they are *John seeks n* and *n is being a unicorn* (where *n* is a notional parameter) and it has no existential import.

Francez's book is literally packed with information; it is, in fact, multiple books in one. It contains a concise introduction into Gentzenian proof theory; it contains an elaboration of the semantic ideas of both Gentzen and the BHK-people, taking them forward into an explicit theory of semantic values; and it contains—and this is the most original part—also an elaborated sketch of PTS for a fragment of natural language, parallel to the celebrated MTS of Montague. Thus it shows that proof-theory is not syntax—at least not in any sense that would prevent it from conferring meanings on expressions. In this way it is, aside of presenting a wealth of new results, usable also as a handbook of structural proof theory. And given that College Publications, who published the book, do not overcharge their customers, buying it is a true deal!

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Proof-Theoretic Semantics, by Nissim Francez. London: College Publications, 2015. Pp. xx + 415.

1 Introduction

After years of study of formal semantics and computational linguistics, Nissim Francez took up the gauntlet thrown down by Schroeder-Heister (Kahle and Schroeder-Heister 2006) regarding the expansion of proof-theoretic semantics (PTS) towards natural language. Series of papers devoted to this issue culminated last year in a whole book, which was published with College Publications. As the author himself states in the preface, 'the book attempts to present as a coherent story both the main line of research pertaining to the PTS in logic, as well as the extension of PTS to NL – an ongoing endeavour' and provide 'reference resource on PTS' (Francez 2015, p. vii).

First, I will go briefly over the contents of the book, then I offer my general assessment.

2 Contents

The book is split into two main thematic blocks dedicated to PTS in logic (roughly 3/5 of the book) and in natural language (2/5 of the book), respectively. These two main parts are accompanied by an opening 'Introduction' section (1/5 of the book), which familiarizes the reader with the general context of the work as well as with the terminology and tools necessary for the rest of the book. Only basic acquaintance with model-theoretic semantics is assumed.

The first part is called 'Proof-Theoretic Semantics in Logic' and it surveys mainly the works of others, but author's new contributions are presented as well. Most notably, the ideas of explicit definition of 'reified' PTS meanings and canonical derivations from open assumptions.

The book covers all the standard PTS topics that the potential reader might be curious about (and even some of the less frequent ones). For example, we can mention different views of PTS (I-view, E-view, …), questions surrounding the PTS notions of harmony (validity, canonicity, assumptions, …), matters concerning appropriate systems (intuitionistic vs. classical), type of rules (natural deduction vs. sequent calculus (and single conclusion vs. multi conclusion), nature of negation (unilateralism vs. bilateralism), etc.

The second part is called 'Proof-Theoretic Semantics for Natural Language' and, as the slight change in title suggests, this is where the majority of novel research of the book is located. As hinted earlier, PTS approaches to natural language are still in their beginnings (despite large strides being done lately, aside from Francez, see also e.g., Więckowski 2011, 2016 or Chatzikyriakidis and Luo 2014). The author hopes that this part will establish 'viability of this approach [PTS] as an alternative to the model-theoretic semantics, in which, so it seems, most linguistics semanticists are entrenched' (Francez 2015, p. vii).

To name the most prominent topics covered, Francez offers PTS account of meaning of sentences with transitive/intransitive verbs and sub-sentential phrases, determiners, intensionality and contextual meaning variation.

3 Opinion

Plentiful references, illustrative examples and overall textbook-like exposition make this book a great resource for both PTS beginners as well as veterans. From this perspective, Francez successfully fulfils the task he set up for himself. Book such as this one was indeed long time needed – it provides a good gateway into PTS for any reader interested in PTS. Perhaps more space could have been dedicated to a discussion regarding the relation between PTS and model-theoretic semantics, but I understand author's choice to keep it brief, as it is probably better left for a more focused publication tailored to this issue.

Restall in his recent review (2016) of *Advances in Proof-Theoretic Semantics* (2016), the first volume in the Trends in Logic series (published with Springer) devoted solely to PTS, wrote:

> The authors of papers in this volume (almost all) agree that proof theory is important to semantics, and they mostly agree that this has something to do with natural deduction and the sequent calculus, but beyond that, there is little agreement on the important problems to tackle or on which framework might best be used to tackle those problems. PTS in this narrower sense is very much a field in flux, and this volume shows it.
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> […]
>
> There are many ideas and approaches on display, with some shared themes and techniques, but there is not so much in the way of coherent and developed methodology. I can only hope there is more to come in this tradition, because there are connections to be made between meaning, use, rules and inference. (Restall 2016)

And while I share many of his points, I think a lot of the things he is looking for could be found in Francez's book. More specifically, with its systematic approach and carefully selected set of tools it offers a uniform framework that can accommodate many of the existing PTS approaches (unilateral, bilateral, I-canonicity, E-canonicity, …) and provide much more (treatment of natural language). This goes together with author's own approach: '…[I] rather view PTS *in a variety of forms* as a universal *definitional tool*' (Francez 2015, p. viii).

But I do not want to reduce this book only to a good PTS handbook. As mentioned before, while the first half book is dedicated mainly to a systematic study of already existing material, it is the second part that contains most of the original research – based almost entirely on the previous work done by the author and his collaborators, namely Dyckhoff and Ben-Avi.

The central notion of 'reified' PTS meaning, upon which Francez's systems stands, does not only allow for an elegant technical treatment of other PTS notions, but also has practical consequences (e.g., compatibility with type logical grammar (TLG)). Although, I imagine that not all inferentialists might be thrilled by the idea of seeing meaning once again as some abstract entity (even though proof-theoretic in its nature). Nevertheless, it is hard to envision how the enterprise of formal semantics can be reasonably carried out without such hypostatized theoretical constructs.

I especially appreciated the parts about the granularity of PTS meaning, demonstrating that Francez's system is truly hyperintensional (by Cresswell's 1985 original specification), since it can distinguish between logically equivalent propositions. At the same time, Francez also proposes simple remedies how to restrain this unbounded hyperintensionality utilizing the notion of grounds of an assertion.

On the other hand, I was missing a PTS treatment of meaning of atomic propositions. As is emphasized on many occasions (see e.g., Francez 2015, p. 196, p. 245, p. 258, p. 259, p. 314), the meaning of atomic proposition is assumed to be given externally to the meaning conferring PTS system. However, the author is well aware of this 'blind spot' and at the very end he frames it into a next open challenge for PTS (more on this later in §4).

Another topic that should be brought up is that philosophical issues remain in the background throughout the whole book. And this is by design and telegraphed by the author at the very beginning:

> There is an attempt in this book to downplay the role of philosophical arguments and disputes, such as realism vs. anti-realism, underlying many of the discussions of PTS, and concentrate instead on the technical details involved in developing PTS. (Francez 2015, p. vii)

Thus, to readers interested in this subject matter, I would recommend to look elsewhere, e.g., Dummett's The Logical Basis of Metaphysics (1991), Brandom's Making It Explicit (1994) or

Articulating Reasons (2000) or Peregrin's Inferentialism (2014). Although these books do not deal with PTS (in Dummett's and Brandom's books it is not mentioned at all and in Peregrin's book it appears only as a short side note), they cover a lot of topics related to its background philosophy and fundamental principles.

Of course, it is an open question how much are these issues even necessary for the advancement of PTS and its successful deployment in practice. That is not to say, however, that all philosophical topics are unimportant, there are still a lot of venues for further investigations (see e.g., the list of open problems of PTS, both technical and philosophical, at the end of this review).

4 Conclusion

The book shows that PTS approaches to natural language are not only feasible, but that they carry along advantages as well. For example, I think there is a lot of untapped potential in regards to its built-it hyperintensionality, i.e., fine-granularity of meaning, which comes very easily to PTS.

Consider e.g., the following Stalnaker's quote:

> …[W]e lack a satisfactory understanding, from any point of view, of what it is to believe that A while disbelieving that B, where the A and the B stand for necessarily equivalent expressions. (Stalnaker 1984, p. 24)

On PTS account, there seems to be no innate mystery: to believe A and not B, while assuming their logical equivalence, is simply to be 'convinced' by the proof of A but not by the proof of B (or more precisely, to be able to recognize the proof of A but not the proof of B). There appears to be nothing puzzling about this position given the varying complexities of proofs or, more generally, justifications.

And speaking of Francez's system specifically, Luo (2014) recently also pointed out the possibility of its embedding in his own PTS-like system (based on Martin-Löf's 1984 constructive type theory). This is certainly another interesting route potentially giving rise to new exciting possibilities.

At the end I would like to point out several directions for further research that Francez proposes throughout his book:

1. PTS treatment of modal operators (Francez 2015, p. 115),

2. relating multiple-conclusion natural-deduction to actual inferential practices (Francez 2015, p. 149),

3. extending PTS towards multi-sentential linguistic constructs (dialogues, discourses) (Francez 2015, p. 322),

4. PTS meaning of atomic sentences (leading to Proof-Theoretic Lexical Semantics) (Francez 2015, p. 377).

Francez explicitly adds the last one to the three challenges for PTS formulated by Schroeder-Heister (Piecha and Schroeder-Heister 2016, p. 253):

5. the nature of hypotheses and the problem of the appropriate format of proofs,

6. the problem of a satisfactory notion of proof-theoretic harmony,

7. the problem of extending methods of PTS beyond logic.

And if we add also those 'unofficial' ones mentioned by Restall (2016), we get:

8. better explore interface and connections between natural language (and real-life deductions) and abstract proof structures,

9. extending PTS beyond the semantics of assertions towards different kinds of speech acts,

10. investigate more connections between meaning, use, rules and inference,

11. examine in more depth how the notions of proof are connected to language use and understanding.

To conclude, PTS has still a long road ahead of itself, as indicated by the above list, but it is also a promising area of novel research, as demonstrated by Francez's book. And I recommend careful reading of both of these.

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