

MATHEMATICAL REVIEWS

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★ **A realist philosophy of mathematics.**

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The epoch of foundationalism in mathematics (1890–1940), represented by logicism, intuitionism, and Hilbert’s program, has now passed. Two different though non-interacting approaches to the philosophy of mathematics have replaced foundationalism. One approach deals with traditional philosophical problems concerned with the epistemology and ontology of mathematics; this approach is adopted mainly in analytic philosophy. However, despite many contributions to mathematics and several clarifications of important philosophical concepts, it has been impossible to reach an objective conclusion concerning the existence and nature of mathematical knowledge. The other approach is historical, where the philosophy of mathematics is explored in relation to examples borrowed from the history of mathematics. In this book, an attempt is made to present a coherent synthesis of these two approaches by advocating the view that mathematics is a quasi-empirical science, that is, mathematical knowledge is in principle fallible. Thus, for example, within what is called “Euclidean geometry”, two different theories can be distinguished, Euclid’s own system and Hilbert’s. The criteria available to decide whether or not a mathematical theory is good have changed in their relative values, so that theories may be incommensurable with one another. Therefore, even though the history of mathematics does not provide a philosophically neutral tribunal before which philosophical questions must be brought, it lies at the heart of an acceptable way of debating the realism/anti-realism issue. It is usually argued that foundationalism is incompatible with quasi-empiricism, but this need not be so if by “foundations” we mean the study of the logic and epistemology of mathematics whose aim is the unification of mathematical theories.

The debate between realists and anti-realists is discussed in terms of a critical account of the various positions from Pythagoras to Dummett and Field. An important case history is related to the infinitesimals. This example shows that the question whether mathematical entities exist is generally inseparable from mathematical practice: the veto against external questions of existence is not tenable. The need arises to provide a context within which not only questions of existence, but any question at all, must be located to have meaning. The realism/anti-realism dispute will not be settled by a formal distinction between internal and external questions of existence or by a correct theory of meaning for the language of mathematics. An original version of realism is then derived from the history of mathematics, which provides support for a non-Platonist form of mathematical structural realism. This is carried out by applying Wittgenstein’s notion of “aspect” to the philosophy of mathematics. Mathematics is a science of patterns, where patterns are neither objects nor properties of objects, but aspects, or aspects of aspects, etc. of concrete objects which become perceptible when a system of representation (mathematical theory) is in place. This view, which lies mid-way between Platonism and anti-realism, explains away how mathematical knowledge is acquired and what the nature of mathematical reality is. Then the ideas of Lakatos are applied to the realism problem. A reconstruction of the internal history of the Cantor-Zermelo set theory makes it clear how mathematical knowledge grows within this theory. Assuming that the question about mathematical realism can be reduced to that concerning whether or not one should be real-

ist with regard to set theory, it turns out that the Cantor-Zermelo set theory is a progressive mathematical research program with strong realist metaphysical assumptions. On the other hand, following Lakatosian terms, constructivism (or intuitionism) with its strong anti-realist metaphysical assumptions is not progressive but degenerative; mathematical constructivism cannot save mathematical objectivity. Therefore, if we accept the Cantor-Zermelo theory, we are committed to a realist view of set theory, a view which extends to the mathematical theories unified by it.

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