Claire Ortiz Hill and Jairo José da Silva. *The Road Not Taken. On Husserl's Philosophy of Logic and Mathematics*. Texts in Philosophy, Vol. 21. London. College Publications, 2013. ISBN 978-1-84890-099-8 (pbk). Pp. xiv + 436.

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Introduction

The Road Not Taken contains 17 chapters (9 by Hill, totaling 190 pages and 8 by da Silva, totaling 177) of previously published essays on Edmund Husserl's philosophy of logic and mathematics and two appendices containing English translations of early letters of recommendation for and assessments of Husserl by (among others) Georg Cantor, David Hilbert, and Hermann Weyl. Behind the book lies its two authors' conviction that Edmund Husserl's early (pre-20th century) philosophy of logic and mathematics remains either largely unknown or, where known, neither appreciated nor appraised properly. It is their shared contention that material published from Husserl's *Nachlass*, especially his research manuscripts on logic and mathematics from the last decade of the 19th century related to the abandoned project of a second volume of *Philosophy of Arithmetic* (hereafter *PA*), together with his few publications from that decade and lectures on logic at the beginning of the 20th century, contain the makings of a philosophy of logic and mathematics superior to that yielded by the analytic tradition in the last century. Presumably the "road not taken" indicated by their book's title, then, in self-conscious reference to the Robert Frost poem (which is reproduced in the book's front matter), is the one that would have led to the development of the material in question. Husserl, however, took another road, that—to continue the allusion to Frost's poem and the apparent interpretation behind it— "has made all the difference."[1]

That difference, of course, involved Husserl's self-appointed mission as the founder of a philosophical science, phenomenology, in which the phenomenological philosophy of logic and mathematics figure as parts, albeit important ones, of the infinite whole whose investigation Husserl assigned to his phenomenology's transcendental self-realization. Hill and da Silva's essays therefore explore Husserl's philosophy of logic and mathematics largely in insolation from Husserl's presentation of transcendental phenomenology. In this regard they silently follow Dallas Willard's precedent of retrieving and reconstructing Husserl's critique of late 19th century philosophies of logic and mathematics independently of Husserl's *Logical Investigations'* (published in 1900) critique of the logical psychologism dominant in many of those philosophies.[2] This is an important virtue of *The Road Not Taken*. Its essays expand the scope of what is treated in Husserl's critique beyond Willard's focus on its argument that the algebraic formalization of deductive logic is not itself a logic but rather a logical technique that, however

1 David Orr, the poetry columnist for the *New York Times* Book Review has recently argued that

> Most readers consider "The Road Not Taken" to be a paean to triumphant self-assertion ("I took the one less traveled by"), but the literal meaning of the poem's own lines seems completely at odds with this interpretation. The poem's speaker tells us he "shall be telling," at some point in the future, of how he took the road less traveled by, yet he has already admitted that the two paths "equally lay / In leaves" and "the passing there / Had worn them really about the same." So the road he will later call less traveled is actually the road *equally* traveled. The two roads are interchangeable. (http://www.theparisreview.org/blog/2015/09/11/the-most-misread-poem-in-america/)

To the extent that the popular interpretation of Frost's poem is implied by Hill and da Silva's choice of their book's title, my critical comments below can be framed within Orr's alternative to the poem's popular interpretation. To anticipate, I argue that Hill and da Silva's assessments of the role of imaginary elements in Husserl's philosophy of mathematics are at odds and that only a phenomenological-constitutional investigation has the philosophical resources to adjudicate their opposing assessments. Both roads, then, were taken by Husserl and have to be taken by those wishing to get to the bottom of his philosophy of logic and mathematics.

2 Cf. Dallas Willard, "Husserl's Critique of Extensionalist Logic: 'A Logic that does not Understand Itself'," *Idealistic Studies* 9 (2) 1979: 143-164, and Dallas Willard, "Husserl on a Logic that Failed," *The Philosophical Review*, LXXXIX, No. 1 (January 1980): 46-64.

expedient, nevertheless remains in principle alienated from the deductive inferences essential to genuine logical progression.[3]

As with most collections generally, the thematic unity of the multitude of essays that compose them is an issue here, compounded in this case by the dual authors and corresponding two sets of essays. Typical strategies to bring about such unity, e.g., a detailed introduction that addresses the contents individually in light of recurrent themes or significant revision of those contents in view of their contribution to an overarching unity were not employed in *The Road Not Taken.* Despite this, however, two general themes gradually emerge, each one roughly specific to each author, such that a division of labor of sorts emerges. Hill's essays tend to focus on Husserl's critique of extensional logic and the consequent superiority of his philosophy of logic over Frege's, because of the latter's—disastrous, on Hill's view, since paradox-inducing—reliance on the extensions of concepts. The essays by da Silva tend to focus on Husserl's philosophy of the logic of symbolic reasoning in mathematics, whose attempt to clarify its nature, scope, and methods da Silva claims (in *The Road Not Taken*'s short Introduction) is superior to Brouwer's and Frege's philosophy of logic and mathematics, which championed respectively the need for "restraint or 'sound' foundations" (xii).

The authors' division of labor issues from their common belief that Husserl's proximity as a student and colleague to such titans of late 19th and early 20th century

3 Topics of this expanded scope include "On Husserl's Mathematical Apprenticeship" (Hill, chapter 1), "Husserl on Geometry and Spatial Representation" (da Silva, chapter 2), "Beyond Leibniz, Husserl's Vindication of Symbolic Mathematics" (da Silva, chapter 3), "Husserl on Axiomatization and Arithmetic" (Hill, chapter 5), "Husserl and Hilbert on Completeness and Imaginary Elements" (da Silva, chapter 6), "The Many Senses of Completeness" (da Silva, chapter 7), "Reference and Paradox" (Hill, chapter 9), "Tackling Three of Frege's Problems: Husserl on Sets and Manifolds (Hill, chapter 10), "On Fundamental Differences between Dependent and Independent Meanings" (Hill, chapter 11), "Incomplete Symbols, Dependent Meanings, and Paradox" (Hill, chapter 12), "Cantor's Paradise, Metaphysics and Husserlian Logic" (Hill, chapter 15), and "Mathematics and the Crisis of Sciences" (da Silva, chapter 17). For a complete list of contents, see *Philosophia Mathematica* III (2014) 22: 139-140.

mathematics as Karl Weierstrass, Leopold Kronecker, Georg Cantor, David Hilbert, and Ernst Zermelo, influenced the development of his thought and indeed his philosophy as significantly as his association with Franz Brentano. Husserl's Ph.D. in mathematics, "Contributions to the Calculus of Variations," was supervised by a former student of Weierstrass's, Leo Königsberger, and Cantor served on the *Habilitationskommittee* for Husserl's *On the Concept of Number.* Not only did Husserl himself credit Weierstrass as the source of "the ethos of his intellectual endeavors" (Hill, 2) but he would also say late in his career "that he had sought to do for philosophy what Weierstrass had done for mathematics" (*Ibid.*). The mathematical point of departure of Husserl's first major work, *PA*, was Weierstrass's conviction that analysis could be rigorized "purely on the basis of positive whole numbers" (*Ibid.*). Despite Husserl's realization before he completed that book that the conviction he had taken over from Weierstrass was unfounded, his response to the problem the discovery of the latter brought to the fore the logical and indeed epistemological problem of the essential scope and limits of symbolic reasoning, which from that point in time on assumed the status of a driving factor in the establishment and development of his phenomenology. Moreover, Husserl's collegial and personal relationships with Cantor at Halle from 1886 to 1901 and Hilbert at Göttingen from 1901 to 1915, placed him in the privileged position of being involved with two of the major sources of the radical reshaping of mathematics that took place at the end of the 19th and beginning of the 20th century.

Regarding the latter, Hill's account of Husserl's response to the paradoxes of set theory he encountered at Halle and Göttingen functions as the cornerstone for her argument that Husserl's philosophy of logic is superior to Frege's and the analytic tradition spawned by the latter's logic. Responding to Husserl's review of Ernst Schröder's *Vorlesungen über die Algebra der Logik*, which criticizes Schröder's claim "that bringing all possible objects of thought into a class gives rise to contradictions" (17), Zermelo communicated to Husserl that "Schröder had been basically right, but his reasoning had been faulty" (*Ibid.*). Hill glosses Husserl's April 16, 1902 "Memorandum of a Verbal Communication from Zermelo to Husserl" (*Husserliana XXII*, 399) as follows: "given a set M which contains each of its sub-sets m, m', as elements, and a set M_o which is the set of all sub-sets M, which do not contain themselves as elements, it can then be shown that M_o both does and does not contain itself" (*Ibid.*). Zermelo's communication referenced page 84 of Husserl's review, which in addition to criticizing Schröder's claim about contradictions connected with the class of all possible objects of thought also contains a significant reservation about calculation with sets. Husserl writes that in cases where we have, "'besides certain classes, also classes *of* those classes, the calculus may not be blindly applied'," (Hill, *Ibid.*), since "[i]n the sense of the calculus of sets as such, any set ceases to have the status of a set as soon as it is considered the element of another set'" (*Ibid.*). And "[i]f one does not keep this in mind, then actual errors in inference can arise" (*Ibid.*).

In fact, "[b]y 1890" (212)—and thus well before his association with Zermelo—"Husserl may have actually been permanently inoculated against recourse to extensions" (*Ibid.*) according to Hill. Husserl's criticism in *PA* of Frege's logical definition of number focuses on Frege's use of the extensions of concepts to establish conceptual identity. Husserl argues that Frege's definition mistakenly infers that from the equivalence, in the sense of one to one correlation, of objects falling under two or more concepts, it follows that the concepts themselves are identical. As Hill represents it, "[a]ccording to Frege's definition, Husserl observes, 'number of Jupiter's moons' would accordingly mean 'having the same number as the concept Jupiter's moons', or more clearly expressed 'having the same number as the aggregate of Jupiter's moons'. Thus one obtains concepts having the same extensions, but not the same intension" (*Ibid*.). In Frege's famous example, the intension of the concept four, which is the number of Jupiter's moons, according to Husserl is different from the intension of "the concept 'any set whatsoever from the equivalence class determined by the aggregate of Jupiter's moons" (*Ibid.*). This holds for Husserl because it can be recognized, without the need for proof, that "[a]ll these sets fall under the number four" (*Ibid.*).

The non-equivalence between concepts and the objects that fall under them argued for by Husserl without proof in *PA* becomes a matter of "essence" in Husserl's unpublished notes on set theory found in Husserl Ms A 1 35 (from 1912, 1918, 1920, 1926 and 1927). These notes "record his reflections about just what the essence, the concept, of set entails" (214). Hill reports, "[i]t is part of the idea (*Idee*) of set to be a unity, a whole comprising certain members as its parts, but doing so in a way that, vis-à-vis its members, it is something new which is first formed by them" (*Ibid.*). Because "[all] mathematico-logical operations performable with sets . . . turn on the idea that sets can be looked upon as kinds of wholes, as new units, formations that are something new vis-à-vis the elements systematized" (*Ibid*.), Husserl maintained that "[i]t would be a contradiction in terms for the system's unity to be one among the elements of the same system" (*Ibid.*). Hill relates both Husserl's critique of extensions and his reflections on the essence of set to Frege's response to Russell's paradox and the contradiction it entailed. She reports, "Frege immediately traced the origin of the contradiction to his axiom of extensionality, Basic Law V of the first volume of *Basic Laws*" (158). Accordingly, it is logically permitted to transform "'a sentence in which mutual subordination is asserted of concepts into a sentence expressing an equality'" (159, Hill quoting Frege). Thus Frege's law would permit logicians "to pass from a concept to its extension, a transformation that, Frege held, could 'only occur by concepts being correlated with objects in such a way that concepts that are mutually subordinate are correlated with the same object'" (*Ibid.*).

According to Hill, Frege would trace the contradiction to the "propensity of language to undermine the reliability of thinking by forming apparent proper names to which no objects correspond" (258). Thus "'[t]he objects that fall under *F* are regarded as a whole, as an object and designated by the name 'set of *F*'s. This is inadmissible because of the essential difference between concept and object, which is indeed covered up in our word languages'" (*Ibid.*, Hill quoting Frege). While Hill does not remark upon Frege's implicit appeal to essences in his account of the "essential difference" concealed by language, she does highlight the role played by the difference between Frege's and Husserl's accounts of the logical issues raised by imaginary numbers in symbolic mathematics. She argues that this difference is behind Frege's need to introduce extensions into his account of the foundations of the theorems of arithmetic. On her view, "Frege *could* not in fact accept combinations of sign (sic) that do not designate an object because his logic was actually designed in such a way as not to be able to cope with them" (205). Frege's logical commitment to objects had its basis in his conviction that "'[o]nly in the case of objects can there be any question of identity (equality)'" (*Ibid.*, Hill quoting Frege). Hill reports that in "'On Formal Theories of Arithmetic' [1885], he argued that unless an equation contained only positive numbers, it no more had a meaning than the position of chess pieces expressed a truth" (*Ibid.*). Thus Hill argues that "[i]t was his need for objects that had induced Frege to introduce the classes, the extensions . . . that he eventually considered to be the cause of Russell's paradox" (*Ibid.*). As Frege himself wrote to Russell, "'But the question is, how do we apprehend logical objects? And I have found no other answer than this, We (sic) apprehend them as extensions of concepts'" (206).

Hill writes, "Husserl also anguished over the logical issues surrounding combinations of signs that do not and cannot refer to objects" (*Ibid.*). However, "[q]uite unlike Frege, Husserl concluded that formal constraints banning reference to non-existent and impossible objects unduly restrict us in our theoretical, deductive work" (207). Husserl's solution to the logical problem presented by the generalization of arithmetic beyond the quantitative domain, namely, of imaginary numbers (including fractions, negative numbers, irrational numbers) "which arithmetically speaking were nonsense" (206), was to show "how reference to impossible objects can be justified" (208). Thus, even "though no object could correspond to what was a contradiction in terms (*Widersinnigkeit*), a contradiction in terms nonetheless genuinely had a coherent meaning and could be determined to be true or false" (*Ibid.*). Husserl himself reports in 1913 that he developed his theory of manifolds in order to solve theoretically the problem of imaginary numbers. It is based on the notion of a complete axiom system, wherein "each grammatically constructed proposition exclusively using the language of this domain was, from the outset, determined to be true or false in virtue of the axioms, i.e., necessarily followed from the axioms (in which case it was true) or did not (in which case it was false)" (*Ibid*.). In such a case, the domain of the manifold is "complete" in a manner that insures "calculating with expressions without references could never lead to contradictions" (*Ibid.*). When two complete manifolds are related to one another "in such a way that the axiom system of one may be a formal limitation of the other" (*Ibid.*), it follows that "all the theorems deducible in the expanded system must exclusively contain concepts that are either valid in terms of the narrower one, and thus not imaginary, or they must contain concepts that are imaginary" (208-209). As an example, Hill reports "Husserl explained, in the arithmetic of cardinal numbers, there are no negative numbers, . . . fractions are meaningless . . . and so are irrational numbers . . . . Yet in practice, all the calculations of the arithmetic of cardinal numbers can be carried out as if the rules governing the operations were unrestrictedly valid and meaningful. One can disregard the limitations imposed in a narrower domain of deduction and act as if the axiom system were a more extended one" (209). Thus, while the "different arithmetics do not have parts in common" (*Ibid*.), they "have an analogous structure. They have forms of operation that are in part alike, but different concepts of operation" (*Ibid.*).

Hill is aware that "[m]uch more can be said about the significance of Husserl's theories about sets and manifolds than can be said here" (216). By focusing on how they developed in response to specific problems, however, Hill is hopeful that what she has written will "help bring Husserl's ideas about sets and manifolds out of the realm of abstract theorizing and prompt further exploration of this philosophical territory, which is as uncharted as it is rich in philosophical implications needing to be drawn and to be made known" (*Ibid.*).

da Silva

Da Silva explores in depth Husserl's account of the logical problems posed by symbolic knowledge in mathematics and the sciences, arguing that "[f]rom the first to the last work he published the task of clarifying the sense and delimiting the scope of symbolization and formalization in science was one of Husserl's major concerns" (62). The logical problems in question all issue from the fundamental epistemological question, "How can we explain that we can obtain knowledge by operating 'blindly' with symbols according to rules, even when these symbols do not represent anything" (61-61)? According to da Silva, "[i]t is unquestionable that Husserl took the epistemological relevance of symbolic presentations for granted" (62), such that without "symbolic reasoning, no science, in particular, no mathematics" would be possible. Husserl's first work, *PA*, formulated the logical problems presented by symbolic knowledge in terms of "'blind' manipulations of meaningful symbols" (64) and "the use of meaningless symbols as if they had a meaning" (*Ibid.*). According to da Silva, "Husserl treated these problems differently." Blind manipulations with numerals and symbolic operations not "presided over by accompanying intuitions" (*Ibid.*) are logically justified because their symbolic system "is an *isomorphic* copy of the system of number concepts and conceptual operations" (*Ibid*.). That is, in Husserl's terminology, they are "*equiform*" (*Ibid.*), in the sense that the symbolic and conceptual domain have a common "*formal structure*" (*Ibid.*). Thus, da Silva maintains, "[w]e can obtain arithmetical *knowledge* by playing with *proper* (i.e. denoting) arithmetical symbols algorithmically only because arithmetical truths are *formal*, i.e., they do not concern numbers strictly, but relations among, operations with, and properties of whatever objects *behave like numbers*" (64-65).

Notwithstanding the formal nature of the system of arithmetical truth for Husserl, da Silva finds it significant that Husserl maintains this system "is articulated internally by a unifying concept—the concept of number as a collection of units upon which we can operate (by inserting or removing units)" (65). On da Silva's view, Husserl's "persistent concern that symbolic systems must be safeguarded from degenerating into dead and dry formalism alien to knowledge, i.e., mere technicalities alienated from *living experiences* (*Erlebniße*) and the *Life-World* (*Lebenswelt*), as he would later say" (*Ibid*), is what is behind his conviction that "a system of formal truths . . . must ultimately refer to a *possible* system of objects unified under a concept whose formal properties the system of truths express" (*Ibid*.). For da Silva's, Husserl's conviction in this regard is, as it were, a two edged sword. On the one hand, it is what allows him to confront more forcefully than his philosophical contemporaries "what I call *the problem of symbolic knowledge*" (66), namely, that of "a proper logical justification for *non-interpreted* symbolic axiomatic systems and *non-denoting* symbols" (*Ibid.*). On the other hand, however, Husserl's insistence that all systems of formal knowledge ultimately have an objective referent will prove in the end to limit unnecessarily both Husserl's philosophical appreciation of the creativity of pure mathematics and his understanding of the way non-classical mathematical physics actually functions.

Already in Husserl's early account of the logical justification of systems of calculation the employment of non-denoting symbols emerged as a problem. To wit, "our usual numeral algorithms cannot do without 0 and 1" (66), while neither symbol denotes number properly understood, that is, number as the answer to the question: how many? Thus 0 and 1 are therefore extrinsic to the concept of quantity under which fall the numbers in the proper sense, i.e., specific determinations of a plurality. In calculation, then, Husserl's early appeal to the isomorphism between the symbolic system and the conceptual domain that it interprets as the logical basis for the knowledge generated by the symbolic system breaks down for the symbols 0 and 1. The logical demand for "the existence of a representational relation based on strict formal identity" (67) behind Husserl's notion of equiformality (isomorphism) is also used in his account of the logical justification of interpreted systems of derivation. The latter concerns "interpreted axiomatic theories, i.e. theories whose axioms are true by virtue of some sort of intuition into what gives the theory its internal unity" (*Ibid.*). The "logical *language* (not simply *calculi*) of such theories must *represent* thinking proper, that is, the formal expressions must stand for meaningful judgments and the formal machinery for drawing conclusions must produce logically sound inferences" (*Ibid.*). However, as in the case of 0 and 1, "a proper *logical* justification for *non-interpreted* symbolic axiomatic systems and *non-denoting* symbols cannot follow along similar lines" (*Ibid.*). In both cases, "of course, we cannot speak of a parallelism between representations and represented, since non-denoting symbols do not represent" (68).

According to da Silva, *PA* only presented a "lame justification" (66) of the logical legitimacy of 0 and 1 in calculation and none whatsoever for imaginary elements (fractions, negative numbers, irrational numbers, etc.) in systems of derivation. (Husserl's unpublished–because never completed–volume two of *PA* was supposed to provide the logical account of the latter.) The wanting justification, soon recognized by Husserl as such, gave the following reasons for "accepting 0 and 1 as numbers" (*Ibid.*): "(1) arithmetical operations among numbers proper produce them . . . and (2) the algorithms for solving numerical problems are worthless without them" (*Ibid.*). In the decade following the publication of *PA*, da Silva identifies earlier and later forms of logically more robust accounts by Husserl to justify cognition in interpreted symbolic systems (theories) that employ imaginary (including 0 and 1) elements in their reasoning. Husserl's earlier account maintained that the theory extended by the introduction of imaginary elements "must be *conservative* with respect to the narrower theory (a fact he confessed to be unable to prove)" (71). His later account "required the narrower theory to be *complete* [definite]–this, of course, implies the conservativeness of the extended theory, provided it is a consistent extension– (a fact that he then thought he knew how to prove, as far as arithmetic is concerned)" (*Ibid.*).[4] Moreover, in addition to Husserl's later account of the logical justification of interpreted symbolic theories, i.e., theories that have as their correlate definite mathematical manifolds, da Silva maintains that Husserl's notion of the pure theory of manifolds envisioned as well a form of logical justification for uninterpreted symbolic formal theories, i.e., theories whose correlates are the forms of mathematical manifolds abstracted from their intended reference or formal extension of definite mathematical manifolds.

4 Because Husserl, according to da Silva, "calls a general statement 'decidable' when its *instances* are decidable" (132), his way of understanding decidability is "certainly not the same one that is behind Kurt Gödel's theorem" (*Ibid.*). Given this difference, da Silva holds that "we should not blame Husserl for making a claim that apparently so blatantly contradicts the famous, although then not proven, Gödel incompleteness theorem" (*Ibid.*). Neither Hill nor da Silva, however, explore the implications of Gödel's theorem for Husserl's theory of complete manifolds.

On da Silva's view, Husserl's early and later accounts of the logical justification of symbolic reasoning nevertheless remain determined by two invariants in his thinking that owed their origin to *PA*. One, that no symbolic calculus can "qualify as a pure theory of deduction, for deductions involve *concepts*, and extensions of concepts cannot determine their concepts, a task only their *contents*, or contents of concepts that are materially equivalent to them, can accomplish" (68). Two, "in order to be logically justified . . . a calculus needs to be adequately correlated with reasoning proper so as to be able to serve as a *substitute* for it" (69). Thus, "[i]f a calculus is logically justified, it then 'stands for' something (even if it can stand for different things), its basic principles and rules are founded on the meaning of what it stands for" (*Ibid*.). Husserl goes beyond his account in *PA* by justifying "the introduction of meaningless symbols" in symbolic systems insofar as "these symbols–no matter how useful from a purely algorithmic perspective–are in the end unnecessary as far as the application of the calculus to its *intended* domain is concerned, despite the fact that their incorporation does not generate formal inconsistencies" (70). This methodological line of logically justifying non-denoting symbols first appears in Husserl's criticism of Schröder's introduction of "0 and 1 in a purely formal way: 0 as the class that can be subsumed under any class, 1 as the class that subsumes any other class" (69). Husserl rejected these purely formal definitions, on the ground that "besides avoiding contradiction [formal contradiction] (*Widerspruch*) . . . a calculus must also avoid *conflict* [falsehood on the intended interpretation] (*Widerstreit*)" (*Ibid*)—the latter understood as "an incongruity between a symbolic system and its *intended* objectual domain" (*Ibid,* n.10). In the case of 0, "Husserl just could not conceive of an empty extension" (69). In that of 1, "[t]he idea of a class that is contained in any other class . . . is absurd, he thought, for there are, after all, disjoint classes" (Ibid.). Because 0 does not denote anything, Husserl "puts it on an equal footing with $\sqrt{-1}$ in general arithmetic" (*Ibid.*).

Husserl's later account of the logical justification of imaginary elements in interpreted symbolic theories goes beyond his earlier methodological focus by introducing the logical notion of the completeness (or terminologically equivalent definiteness) of the conceptual (narrower) theory and stipulating that the extended theory be consistent with the narrower. Husserl thought this establishes the logical conditions for treating imaginary elements "like real ones" (73). This logical justification, however, is limited, insofar as in its formulation of the use of imaginaries they are presented as contributing nothing to the knowledge of the contents of the conceptual theory. As da Silva puts it, "Husserl insists that as long as we are interested in knowing the properties (even only the formal properties) of the *concept* that founds a theory (for instance, the properties of numbers as *numbers strictu sensu*), the use of imaginaries cannot be an essential one" (*Ibid.*). This limitation of symbolic reasoning maintained by Husserl in the case of interpreted theories likewise holds for the uninterpreted symbolic theories composing the pure theory of manifolds. As da Silva presents it, "[a]lthough . . . purely symbolic theories are *per se* a form of knowledge, structural or formal knowledge precisely–they provide knowledge of formal manifolds independently of their interpretations, thus belonging to formal ontology–, they must be teleologically oriented towards objectual domains" (*Ibid.*). Because "formal theories are mere *forms of theories* . . ." (*Ibid.*), da Silva maintains that for Husserl "the creation and study of formal theories for their own sake, independently of intended applications, amounts to toying with what we can call 'formalist alienation'" (73-74). And this despite the fact that "Husserl sees even formal theories as referring to objects, *formal objects* precisely, indeterminate as to content, but determinate as to form by their theory" (73, n.17).

Da Silva's account of Husserl's logical justification for symbolic reasoning is driven by his critical concern about Husserl's inability to "see a calculus as a free creation" (70). For da Silva, this is what was behind Husserl's unnecessarily "cautious treatment of purely symbolic knowledge" (75). Regarding imaginaries, da Silva has "more serious concerns" (76), as he believes "Husserl was so worried about securing mathematics against a possible infection with imaginaries that he put more effort into developing a protective vaccine than into explaining *why* imaginaries are useful when they are . . . . Husserl believed that *a priori* mathematical contentual theories are in general *conceptual* theories and that imaginaries cannot substitute relevant intuition and be *essentially* involved in the business of proving theorems" (*Ibid.*). However, da Silva argues that "there is not much difference between contentual or conceptual, on the one hand, and purely symbolic mathematical theories, on the other: theories of both types are in a sense formal, since their objects are only and invariably *forms* or *structures*" (*Ibid.*). Indeed, "[t]he fact that even contentual mathematics is a formal science reduces Husserl's distinction between theories and mere forms of theories to one between conceptual or eidetic formal theories (such as arithmetic and physical geometry) and hypothetical formal theories (such as Riemannian n-dimensional geometries)" (*Ibid.*). On da Silva's view, then, Husserl keeps them apart because of his concern for "epistemological relevance: the former are already theories of something, the latter only describe possible hypothetical forms waiting for objectual domains to appear that can be in-formed by them” (*Ibid.*).

Critical Considerations

Hill’s and da Silva’s differing assessments of Husserl’s theory of manifolds raise the issue of their appropriate philosophical adjudication. Because they both recognize the mathematical-logical problem of imaginary elements in arithmetic as what was behind the theory and are in accord regarding the crucial roles of the completeness of the narrower theory and the consistency of the extended theory, the difference at issue is not a matter of interpretation. In arguing for the superiority of Husserl’s theory over Frege’s by stressing that unlike Frege, Husserl’s theory is free of the commitment to the logical objects yielded by the extension of concepts that unnecessarily restricts Frege’s logical account of imaginaries, Hill singles out for praise the focal point of da Silva’s criticism. She does so insofar as the latter maintains that Husserl unnecessarily restricted mathematical manifolds by conceiving them essentially as having a teleological intention toward objectuality. Of course, Husserl’s theory’s formulation of logical objects may be both less restricted than Frege’s but still restricted in the sense maintained by da Silva, which renders all the more pertinent the question of how to adjudicate philosophically what is at stake here. One way it cannot be done, I would submit, is by taking as its point of departure the foundational concepts and methodological presuppositions of the “road not taken” that Hill’s and da Silva’s essays explicate and analyze so well. Their differing assessments of Husserl’s theory of manifolds provides sufficient evidence that another road must be taken.

I propose that the road that Husserl actually took, or better, the transcendentally critical transformation of Husserl's philosophy behind it, provides the most appropriate point of departure for adjudicating these matters. Inseparable from this transformation is the problem of transcendental constitution. Husserl himself in 1929 relates it to his first book, which "in spite of its immaturity" (*FTL*, 76/86), was "in my later terminology" (*Ibid.*, 76/87) both "a phenomenologico-constitutional investigation" (*Ibid*) and "the first investigation that sought to make 'categorial objectivities' of the first level and of higher levels (sets and cardinal numbers of a higher ordinal level) understandable on the basis of the 'constituting' intentional activities, as whose productions they make their appearance originater, accordingly with full originality of their sense" (*Ibid.*). Of course, that first book's approach understood these activities in terms of their non-causal, descriptively characterized psychological genesis, Husserl's abandonment of which was coincident with his development of a descriptive phenomenology culminating in a transcendental phenomenology. Despite, however, Husserl's remarks in 1929, he himself never revisited in detail his earlier investigations from the standpoint of the phenomenological investigation of constituting intentional activities.

An investigation from this standpoint would have to address the issue of the constitution of what *PA* characterizes as "the distinction between *symbolic* and *authentic* [eigentlichen] presentations of number [*Zahlvorstellungen*]" (190/200). Or better, removing the psychological language of this immature work, the distinction between symbolic and authentic concepts of number would have to be addressed. Both Hill and da Silva show that for Husserl the problem of non-referring or non-denoting symbolic reasoning enters into mathematics with the introduction of imaginary elements. However, it does not follow from this that all non-referring or non-denoting, that is, symbolic, components of mathematics are imaginary. Hill and da Silva likewise both recognize this. Hill, on the one hand, does so indirectly (99), insofar her discussion of the first law of arithmetic according to Husserl, i.e., a+b=b+a, involves an equation whose symbols do not have an imaginary meaning. Da Silva, on other hand, is more direct, as he straightaway understands the arithmetic manipulation of numerals to be on Husserl's view a part of symbolic reasoning requiring logical justification. The non-imaginary but nevertheless symbolic status of numerals and elementary variables raises the question of their notational or referential status. Imaginary elements do not denote and therefore do not refer by definition, since their status as imaginary is constituted by their lack of meaning when measured by the logical meaning intrinsic to the system of natural numbers. But what about the numerals that blindly "stand for" these numbers and the letter signs that blindly stand for any arbitrary number? How are they constituted?

Hill would most likely address this question by following Husserl's critique of Frege's employment of extensions to provide the logical foundation of number, and therefore rule out the numeral (number sign) standing for a set of logical objects. Because Husserl's critique argues that an extension and number do not form an identity, assigning a number sign to a set taken as a whole apart from the objects that compose its extension, would in essence mistake a concept, i.e., the set, for one of its objects (elements) or otherwise violate their essential difference. But so far as I can tell, Hill's faithful presentation of Husserl's view in 1902/03, "that the laws of arithmetic just unfold what is found in the concept of number" (107), that their "pronouncements" (*Ibid.*) "are just about numbers" (*Ibid.*), doesn't address the issue of *how* the non-denoting symbols that make these pronouncements are themselves constituted.

Da Silva would most likely address the question of the constitution of the symbolic function of the number sign by appealing to his elaboration of Husserl's account of the isomorphism between the sign functioning as a numeral (the symbolic presentation of number in the argot of *PA*) and the number concept (the authentic presentation of number in *PA's* argot) that is the source of the logical justification of blind manipulations with numerals nevertheless yielding knowledge about numbers. However, because as we've seen, da Silva refines Husserl's account, numerals then wouldn't be constituted as isomorphic *copies* of number concepts per se, but rather, because of the formal nature of arithmetical truth, they would be isomorphic with "the relations among, operations with, and properties of whatever objects *behave like numbers*" (65). But again, so far as I can tell, this account doesn't address the issue of *how* a letter sign is constituted as an isomorphic copy of the formal structure of whatever objects behave like numbers, let alone how such number like objects themselves are constituted. Da Silva, of course, has a ready response to this concern, namely that its very formulation is problematical, given that in the end there's "not much difference" (76) between conceptual or contentual theories and pure symbolic mathematical theories. The vanishing difference being rooted in the fact that the objects of both types of theories "are only and invariably *forms* or *structures*" (*Ibid.*). From the standpoint of a phenomenologico-constitutive analysis, however, an account of the *how* at stake here would require that the constitution of the formality of formal objects common to conceptual and pure symbolic mathematics be made evidently prominent and articulated descriptively.

Neither Hill nor da Silva should be blamed for not addressing these constitutional issues but rather praised for their presentation of a collection of essays that treated individually make the case for the contemporary importance of Husserl's philosophy of mathematics and logic; and that, considered collectively, can be seen as making salient the need for phenomenological-constitutional analyses to adjudicate the fundamental problems that emerge as being behind their arguments for that importance. But someone needs to be held responsible for the preponderance of typographical errors that mar this volume. I counted over three dozen, a number of which challenge the reader's understanding as they involve misplaced or missing negatives.