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Smoryński, Craig
★Adventures in formalism.
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The use of suitable formalisms is a common tool in mathematics, and from the outside wrongly often seen as characteristic of mathematics. Nevertheless, good formalisms occasionally make using and doing mathematics much easier.

The author of this book offers a historically oriented discussion of the use of formalisms in mathematics. He distinguishes three types of formalisms: (i) more or less heuristic reasoning combined with calculations of a certain sort, as recognizable, e.g., in the work of Euler and in the use of $\sqrt{-1}$ in late mediaeval times; (ii) a strong precisification of the involved mathematical notions and clear and gapless chains of arguments as in the work of Cauchy, or Weierstrass; and (iii) the full formalization of mathematical theories as discussed using formalized languages and modern mathematical logic.

In a nutshell, in Chapter I, "Infinite series" (pp. 1–86), these different types of formalisms are exemplified through the discussion of infinite series. Chapter II, "Classic examples of formalism" (pp. 87–190), is devoted to formalism in the first sense, and ranges from Euclid via Galilei and Taylor to Euler and Dirac. Chapter III, "New numbers from old" (pp. 191–448), discusses the formalism in the second sense through the topic of the successive expansion of the number systems from the introduction of the negative integers to the introduction of the non-standard reals, and it ends with the topic of continuous but non-differentiable functions. Finally, under the heading "The axiomatic method" Chapter IV (pp. 449–556) presents the formalism in the third sense, and is accompanied by the closing Chapter V, "The crisis of intuition" (pp. 557–594), which discusses critical reactions to the use of those diverse types of formalism in mathematics.

The diverse mathematical material is well presented without larger gaps, and hence easily accessible to graduate students and mathematics teachers. The book, however, is of interest to everyone interested in the history and philosophy of mathematics.

Siegfried Gottwald