How deontic logic ought to be: towards a many-sorted framework for normative reasoning

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Abstract

Formalising adequately normative logical reasoning with deontic logic has been notoriously problematic. Here I argue that one of the major reasons is that a typical deontic inference combines different types of sentences, expressing (inter alia) propositions, norms, and actions. These have different logical properties and formally mixing them can leads to unnatural (or, plainly absurd) conclusions, of which deontic logic abounds. Thus, I argue that deontic logical reasoning is inherently many-sorted and that an adequate logical formalisation of such reasoning ought to involve separate, yet inter-related syntactic sorts, at least including *norms, actions, and propositions*. Here I propose such formal logical framework, illustrate its use for formalising commonsense normative reasoning, and provide formal semantics for a large fragment of it.

Keywords: logic-based normative reasoning, many-sorted deontic logic, actions, norms, propositions

1 Introduction

The questions of how logic-based normative reasoning should be formalised, and what it should apply to, have permeated the entire history of deontic logic and have been driving much of its agenda, ever since (and even before) G.H. von Wright's seminal 1951 paper [38]. Von Wright himself struggled with these questions for over 50 years and changed his views and opinions more than once meanwhile. His original system of deontic logic proposed in [38] was not a logic of propositions, but a logic of norms over actions ('acts') [2], to which the deontic operators apply, thus producing normative propositions. That led to various problems, both formal and conceptual, which von Wright tried hard to resolve over the following years, meanwhile gradually moving towards the (so called) 'standard deontic logic' of propositions, cf. [40,41,42]. This was a

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[2] Von Wright wrote there "*First a preliminary question must be settled. What are the "things" which are pronounced obligatory, permitted, forbidden, etc.? We shall call these "things" acts.*"

shift, in von Wright's terms, from an *ought-to-do* ("Tun-sollen") approach to an *ought-to-be* ("Sein-sollen") approach to Deontic logic (cf. [45]). That shift was also motivated by the vigorous development of modal logic in the 1960s and von Wright (as he himself admits, e.g. in [46]) was strongly influenced by some leading logicians of the time and proponents of the possible worlds semantics, incl. Anderson, Prior, and others.

The so-called 'standard' propositional deontic logics emerging as a result of that shift not only did not resolve the fundamental problems of logical formalisation of normative reasoning, but actually aggravated some of them, by bringing to the surface numerous formalised versions of deontic paradoxes and puzzles, such as Ross' paradoxes, Prior's paradoxes of derived obligations, etc. Much of the mainstream research in deontic logic has been devoted to attempting to resolve these paradoxes, either one at a time, or "*all of them in one fell swoop*" [4]. Indeed, much progress has been made over the years, but also many problems arising in the area have not been resolved yet in a satisfactory way, and more have arisen meanwhile. In particular, the fundamental question "*(how) is formal logic applicable to normative reasoning?*" remains, I would argue, not definitively resolved yet. Just one very telling fact about the long-lasting drama around that question is that 40 years after his original 1951 paper in *Mind* von Wright published in 1991 the paper [44] titled "*Is there a logic of norms?.*" In the abstract he wrote "*If norms are neither true nor false, can logical relations such as contradiction and entailment obtain between them? Earlier logical positivists [...] have answered the question with No. While appreciating the seriousness of the problem, the author of the present paper makes a fresh attempt to answer the question with Yes. [...]*"

Arguably, most of the problems with formalising normative reasoning are inherent in its very nature, which combines usual propositional reasoning with reasoning about norms, about agents' actions, and judgements about the agents' compliance with norms while performing these actions. It is also widely acknowledged that it is quite challenging (if possible at all) to capture all these in 'traditional' logical systems, involving a single sort of formal expressions, viz. formulae expressing propositions. Numerous attempts have been made to develop more elaborated such systems that would capture better the essence of normative reasoning. In particular, several systems of deontic logic have been proposed (see notes on related work in Section 6) putting together actions and norms, or actions and normative propositions. Still, it appears that these attempts have not yet led to a full and seamless integration of various deontic logics of norms, logics of propositions, and logics of actions, into a *deontic logic combining reasoning about norms, propositions and actions on a par*. This, in a nutshell, is the essence of this paper's proposal.

Here I argue that such more elaborated approach is not optional, but necessary for the design of adequate logical systems for normative reasoning. In particular, I claim that adequate logical formalisation of normative reasoning ought to be *many-sorted*, involving separate syntactic sorts, at least for *norms, actions, and propositions.* Then I propose a concrete, yet generic such new logical framework, for which I present here the basic building blocks of its language, provide formal semantics for a large fragment of it, and illustrate its use for formalising 'everyday' normative reasoning.

2 Some fundamental issues of deontic logic revisited

2.1 Jørgensen's dilemma: is there a deontic logical consequence?

As noted in the introduction, a fundamental question arising even before the birth of formal deontic logic is "*What do deontic sentences express: norms or propositions?*" It has been widely assumed that these two uses are mutually exclusive. The traditional logical positivism view was definite: deontic sentences expressing norms can be neither true nor false, so they are not propositions and cannot be treated with formal logic. One of the leading representatives of that view, the Danish philosopher J. Jørgensen, stated that issue – now known as *Jørgensen's dilemma* – in his 1937 paper [20] essentially as follows:

- *either* the notion of logical consequence is defined in terms of truth, in which case there can be no deontic logical consequence, hence no possibility for deontic logic;
- *or* a logic of norms is possible, but the notion of logical consequence should not be defined in terms of truth, which contradicts a fundamental assumption in logic (according to the logical positivism).

Von Wright's position (cf. e.g. [39]) was strongly in favour of the latter: "*Yes, logic of norms is possible, as logic has a wider reach than truth!*".

Let me also note that the question whether there can be a coherent notion of respective logical consequence is not exclusive to normative reasoning. Such questions arise, on similar grounds, for instance regarding reasoning about imperatives, as well as about interrogatives [3].

2.2 Norms vs normative propositions

One of the fundamental issues arising in normative reasoning is the important distinction that is to be made between a *norm* and a *normative proposition*, perhaps first explicitly pointed out (according to von Wright) by I. Hedenius [17]. What makes this distinction so subtle is that it is often noted only on level of pragmatics. For instance, a normative sentence such as "*Parking here is forbidden*" may have a *prescriptive (norming) meaning*, if stated by an authority (parking attendant or traffic police), or a *descriptive (informative) meaning*, if uttered by a possibly informed passer-by. The former case yields a *norm*, whereas the latter – a *(normative) proposition.* That makes a major difference, as norms prescribe what should, or may (or not) be done, whereas normative propositions describe the normative status of actions according to the existing norms. Thus, norms can be obeyed, fulfilled or violated, but *cannot be true or*

[3] However, it should also be noted that there has been a significant recent progress on the latter issue, by means of the so called inquisitive semantics and logics, cf. [6]. Compared to that development, formalising logical normative reasoning is now lagging behind.

false, whereas normative propositions are naturally assigned truth values.

According to von Wright, the appearance of many paradoxes (such as Ross' letter paradox) arises from a confusion between norms and norm-propositions, as norms do not satisfy some basic logical laws, e.g.:
"*A norm to the effect of 'p' does not imply a norm to effect that 'p or q'*".

Several scholars on normative reasoning, incl. C. Alchourrón and E. Bulygin [2], kept raising the question *Is Deontic Logic a logic of norms or a logic of normative propositions?*. According to the early von Wright, the 'real deontic logic' is the former. But, as noted earlier, he changed his views more than once over time, cf. [43], and later he took the view that logic of norms is impossible and deontic logic can only be a logic of propositions about (the existence of) norms [4] – from which he backtracked again still later, cf. e.g. [46] [5].

The answer advocated in the present work is that Deontic Logic ought to be *both* a logic of norms *and* a logic of normative propositions. Also, in my view, Deontic Logic should combine the *Sein-sollen* and the *Tun-sollen* perspectives [6], rather than opposing them to each other.

2.3 Can there be formal logical deontic reasoning? Yes, there can, and there is!

Jørgensen's dilemma raises the existential for deontic logic question: *can there be deontic logical consequence, at all?*. As already noted, von Wright's position, albeit sometimes shrouded in doubts, was positive. This view is shared by most (if not all) formal deontic logicians, and even by those who view the so called 'standard systems of deontic logic' to be a failed attempt to adequately formalise deontic reasoning. For the present author, the question is but rhetoric, because deontic logical reasoning *does exist in real life* and we do make deontic logical inferences on a daily basis, even though usually without realising that. The real question is: *how to formalise properly deontic logical reasoning*? In several papers von Wright explored the notions of consistency and entailment between norms, but (for all I know) stopped short of putting them on a par with normative propositions. As I argued above, a major problem arising with normative logical reasoning is that it is intrinsically many-sorted, combining propositions, norms, actions, and only if formalised as such it can be captured

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[4] As he confessed in [44]: "*Over the years my view became more "radical", and I came to think that logical relations such as contradiction and entailment could not hold between (genuine) norms and that therefore, in a sense, there could be no such thing as a "logic of norms."* " [...] "*The notion of rationality came to my help and so I arrived at a position according to which deontic logic is neither a logic of norms nor a logic of norm-propositions but a study of conditions which must be satisfied in rational norm-giving activity*".

[5] von Wright adds there in a footnote: "*This was what I thought initially to be the lesson of the coming into existence of deontic logic. Later I thought differently. In the end, it seems, I have gone full circle back to my original position. But I still think the journey was worth making.*"

[6] This alternative is distinct, but in my view essentially related to the previous one, as deontic logic of norms links more naturally with the *ought-to-do* approach, whereas a logic of normative propositions is closer to the *ought-to-be* perspective.

adequately.

Let us consider a few simple examples of what one can call 'everyday normative reasoning' to see the issue at hand.

(i) *Pippi is buying beer from the local liquor store.* (action; fact)

(ii) *Everyone*[7] *of age over 20 is allowed to buy alcohol from liquor stores.* (norm (permission))

(iii) *Pippi is 21 years old.* (fact)

(iv) *Therefore, Pippi is allowed to buy alcohol at liquor stores.* (conclusion: derived individual norm? normative proposition? both?)

(v) *Therefore, Pippi's buying beer from the local liquor store is legal.* (conclusion: a normative proposition, stating compliance with a norm)

Another example:

(i) *Smoking in the building is forbidden.* (common norm (prohibition))

(ii) *Chuck Norris is in the building.* (fact)

(iii) *Therefore, Chuck Norris is not allowed to smoke.* (derived individual norm (prohibition))

(iv) *Chuck Norris is smoking.* (action; fact)

(v) *Therefore, Chuck Norris is violating the non-smoking rule.* (fact; normative proposition, stating a norm violation)

(vi) *Chuck Norris may violate any rule.* (???... ok, a Chuck Norris joke)

Clearly, these are proper logical inferences (skipping a few trivial steps). Yet, they are *not* instances of traditional logical reasoning, because they involve *not only propositions* (facts), but also actions and (common and individual) norms, and these do not necessarily obey the rules of propositional reasoning, at least because they cannot be assigned truth values. Still, we should all agree that these inferences are – intuitively – *logically correct*, in a sense that ought to be made precise.

The logical inference problems get amplified when *conditional* norms are involved, often leading to non-monotonic reasoning, as in the following example.

(i) *To be allowed to drive a car, one must have a valid driving licence.* (norm (conditional obligation))

(ii) *Noone under the age of 18 may have a driving licence.*

[7] This sentence formally requires universal quantification over agents. However, a language with full-fledged quantification over agents is hardly necessary for normative reasoning, as norms usually involve universal quantification over agents and very seldom existential quantification (e.g. as in "at least one author of each accepted paper must register for the conference"). The universal quantification over agents can thus be omitted and assumed implicitly. So, to avoid such quantification and keep the arguments propositional, I will use in the formal framework so called 'common norms', applying by default to all agents.

(norm (prohibition))

(iii) *Tommy is 16 years old.* (fact)

(iv) *Therefore, Tommy is not allowed to drive a car.* (conclusion 1: a norm and a normative proposition)

(v) *A person aged between 16 and 18 may practice driving, but only if accompanied by a licensed driver.* (norm (conditional permission))

(vi) *So, Tommy may drive a car, if accompanied by a licensed driver.* (conclusion 2: a conditional norm, and a normative proposition)

(vii) *Noone is allowed to drive under the influence of alcohol.* (norm (prohibition))

(viii) *Tommy has just had two beers and is a little dizzy.* (fact)

(ix) *Therefore, Tommy is not allowed to drive a car (now)*[8] . (conclusion 3: a norm and a normative proposition)

(x) *Tommy is driving his dad's Volvo back home.* (action; fact; norm violation)

Note that some of the expressions in the inference above can be classified in more than one way, e.g. the last one is an action, but also a fact, and a proposition implying a violation of the prohibition norm stated in (g). In order to treat properly these type-sharing phenomena we need syntactic mechanisms that generically transform one type of statement into another.

3 MS-Deon: a multi-sorted logical language for normative reasoning

As discussed earlier, initially von Wright conceived deontic logic as a logical theory of *ought-to-do*. However, he and others gradually transformed it into a logical theory of *ought-to-be*. I argue that neither of these approaches suffices alone to capture the distinction and interaction between *actions, norms and normative propositions*, but they must be combined in a more sophisticated and versatile language, with different syntactic sorts of formal expressions, involving (at least) each of these. Furthermore, *agentivity* should be brought to the fore of normative reasoning, as norms should (primarily) apply to agents and their actions, not to propositions, i.e. von Wright's original idea was the right one! Importantly, *existence and validity of norms* should be separated already on syntactic level from statements about *agents' compliance with norms*, as truth values apply to the latter, but not the former. Eventually, both validity and compliance with norms should be postulated for the basic norms, and then *computed, or deductively derived* for complex norms.

Before introducing the formal language, here are some guiding principles followed in its construction.

[8] This example raises more issues, such as the role of time and temporality in normative reasoning, which I will not discuss further here.

- *Norms* are constructs applying deontic operators D (*Obligatory, Permitted, Forbidden*) to actions, to produce *D-to-do* expressions. Norms can be applied to actions of specific agents, or universally, to all agents. The latter can be done generically ("smoking is forbidden") or by explicit quantification ("no passenger is allowed to smoke"). I will follow the former approach (cf. footnote 7). Furthermore, often norms are *conditional* (e.g. "persons of age below 18 are not allowed to buy alcohol").
- Norms can be combined by boolean and other operations to produce complex norms. More generally, a *norm-building sub-language* emerges.
- *Actions* can be atomic, primitive entities, or can be built in a compositional style of dynamic logic, from (names of) atomic actions, by using operations on actions. Thus, an *action-building sub-language* emerges, too.
- Actions also involve specific STIT-like constructions, where only the required effect of the action, and possibly the agent executing that action, are specified, thus relating *D-to-be* and *D-to-do* expressions.
- *Propositions* can be of different sub-sorts, too, including *factual, normative*, and *performative* (explained further). They are built separately, by imposing suitable restrictions on the language, but can be eventually combined by applying standard logical connectives.

3.1 The formal 3-sorted deontic language MS-Deon

Here I present a generic multi-sorted language of **Deontic Logic for Norms, Actions, and Propositions**, hereafter denoted MS-Deon, with the three main sorts defined by mutual recursion, along with the intuitive semantics of the respective language constructs. This language will generically involve various possible constructs and sub-sorts, but these need not all be included in any concrete instantiation of it, which would only select the constructs that are naturally needed for the concrete purpose of specific normative reasoning.

To define the language, I first fix sets of atomic propositions PROP, atomic actions ACT, and agents Agt.These will usually (though, not necessarily) be assumed finite and common for all formal models that will be defined further and can be thought as specifying the signature of a concrete instantiation of MS-Deon, analogously to the non-logical symbols of a concrete first-order language. I will use specific names for agents, for which I will use metavariables such as $\text{A}, \text{B}, \text{C}$. I also use *agent parameters*, i.e. free variables ranging over agents; typically denoted by a, b, c. These parameters will only play an auxiliary role, to avoid explicit universal quantification over agents (cf. footnote 7) which is typically needed for common or conditional norms.

- **Actions.** The sort for actions is built compositionally, generally following the style of Propositional Dynamic Logic PDL, by this inductive definition:

$$\alpha := \alpha_{at} \mid \bar{\alpha} \mid \alpha;\alpha \mid \alpha \cup \alpha \mid \alpha \cap \alpha \mid \cdots \mid \varphi? \mid [\text{stit}]\varphi$$

Here α_{at} are atomic actions, $\bar{\ }, ;, \cup, \cap$ are respectively the operations of 'negation', sequential composition, choice, and parallel execution of actions, whereas $\varphi?$ is the action 'test' applied to a formula φ. Intuitively, the action $\varphi?$ succeeds without changing the current state if φ is true in it, else it fails, leading to no outcome state ('crashes'). Lastly, $[\text{stit}]\varphi$ is an action [9] which is only described by its effect, viz. 'bringing about (the truth of) φ'. Note that actions are so far agent-less, but in the context of the full language they will be usually attributed to agents.

- **Norms.** I consider three sub-sorts of norms: *common* (applicable generically to any agent), *individual agentive* (applicable to an explicitly specified agent), and *conditional agentive* (applicable to all agents satisfying the given condition). In the definitions below: α is an action; $\varphi(\mathsf{a})$ is a proposition (possibly) containing an agent parameter a; A is (a generic name of) an agent; a is a parameter ranging over agents; $\mathsf{Ought}^{do}, \mathsf{Perm}^{do}, \mathsf{Proh}^{do}$ are the main deontic to-do operators, viz. 'ought-to-do', 'permitted-to-do', and 'prohibited-to-do'; and $\bar{\ }, \bigcirc\!\!\!\!\vee, \bigcirc\!\!\!\!\wedge$ are the respective boolean operations on norms, the precise meaning of which can vary (cf. e.g. [40,43,32,35,36] for possible interpretations) and will not be fixed here. Formally:

  Common norms:

  $$\mathsf{N} := \mathsf{Ought}^{do}\alpha \,|\, \mathsf{Perm}^{do}\alpha \,|\, \mathsf{Proh}^{do}\alpha \,|\, \bar{\mathsf{N}} \mid \mathsf{N} \ovee \mathsf{N} \mid \mathsf{N} \owedge \mathsf{N}$$

  Individual agentive norms:

  $$\mathsf{N} := \mathsf{Ought}^{do}_{\text{A}}\alpha \,|\, \mathsf{Perm}^{do}_{\text{A}}\alpha \,|\, \mathsf{Proh}^{do}_{\text{A}}\alpha \,|\, \bar{\mathsf{N}} \mid \mathsf{N} \ovee \mathsf{N} \mid \mathsf{N} \owedge \mathsf{N}$$

  Conditional agentive norms:

  $$\mathsf{N} := \mathsf{Ought}^{do}(\varphi(\mathsf{a}), \alpha) \,|\, \mathsf{Perm}^{do}(\varphi(\mathsf{a}), \alpha) \,|\, \mathsf{Proh}^{do}(\varphi(\mathsf{a}), \alpha) \,|\, \bar{\mathsf{N}} \mid \mathsf{N} \ovee \mathsf{N} \mid \mathsf{N} \owedge \mathsf{N}$$

  Intuitively, $\mathsf{Ought}^{do}(\varphi(\mathsf{a}), \alpha)$ says that doing α is obligatory for every agent a satisfying the condition $\varphi(\mathsf{a})$; likewise for Perm^{do} and Proh^{do}.

  These sub-sorts of norms can be combined further by using the boolean operations on norms.

- **Propositions.** I consider 3 most sub-sorts of propositions that naturally and most commonly occur in normative reasoning.

  Factual propositions:

  $$\phi := p \mid p_{\text{A}} \mid p_{\mathsf{a}} \mid \neg\phi \mid \phi \wedge \phi \mid [\alpha]_{\text{A}}\varphi \mid [\alpha]_{\mathsf{a}}\varphi \mid [\alpha]\varphi$$

  Factual propositions are built up from atomic propositions to describe facts of the world. They can be agent-less, or referring to a specific agent (p_{A}, meaning that the agent A satisfies a property p), or agent-parameterised (p_{a}, with likewise meaning). Factual propositions can also express statements

[9] NB: even though the idea comes from STIT theories, there is a fundamental difference: here $[\text{stit}]\varphi$ is an action, not a formula.

about facts holding after executions of actions, e.g. $[\alpha]_{\text{A}}\varphi$ meaning to say that 'after the agent A executes the action α, (the fact expressed by) φ will hold'; likewise for $[\alpha]_{\mathsf{a}}\varphi$; respectively, $[\alpha]\varphi$ means 'after any execution of the action α, φ holds'.

Performative propositions:

$$\psi := \mathsf{Perform}_{\text{A}}(\alpha) \mid \mathsf{Perform}_{\mathsf{a}}(\alpha) \mid \mathsf{Perform}(\alpha) \mid \neg\psi \mid \psi \wedge \psi$$

Performative propositions express claims about actions being performed, by specific agents, or in general. Optionally, they can also involve propositions expressing *abilities* of agents to perform actions, e.g. $\mathsf{Able}_{\text{A}}\alpha$, etc., as well as other attitudes towards actions (desires, intensions, etc.).

Normative propositions:

$$\theta := \mathtt{InForce}(\mathsf{N}) \mid \mathtt{Sat}_{\text{A}}(\mathsf{N}) \mid \mathtt{Sat}(\mathsf{N}) \mid \mathtt{Legal}_{\text{A}}(\alpha) \mid \neg\theta \mid \theta \wedge \theta$$

Normative propositions express claims about the validity of norms, where $\mathtt{InForce}(N)$ means to say that "the norm N is (currently) 'in-force' ", as well as claims about the compliance or violation of norms, by specific agents or generally. For instance, $\mathtt{Sat}_{\text{A}}(\mathsf{N})$ means to say that 'the agent A is complying with the norm N ' and $\mathtt{Legal}_{\text{A}}(\alpha)$ says that 'the performance of action α by agent A legal (norm-compliant)'.

Inter-sort propositions can be combined freely by applying boolean connectives to these sorted propositions.

3.2 Some notes on the expressiveness and use of MS-Deon

Inter-sort transitions. MS-Deon allows for seamless transitions from one sort to another, whenever that makes good sense. For instance, an action α transforms to norms $\mathsf{Ought}^{do}\alpha, \mathsf{Ought}^{do}_{\text{A}}\alpha$ and likewise to $\mathsf{Perm}^{do}\alpha$ and $\mathsf{Proh}^{do}\alpha$, as well as to propositions like $\mathsf{Perform}(\alpha)$. A norm N transforms to normative propositions $\mathtt{InForce}(\mathsf{N})$ and $\mathtt{Sat}(\mathsf{N})$, whereas a normative proposition $\mathtt{InForce}(\mathsf{N})$ can be tested with a test action $\mathtt{InForce}(\mathsf{N})?$. Further, $\mathtt{Sat}(\mathsf{N})$ can be transformed to an action $[\text{stit}]\,\mathtt{Sat}(\mathsf{N})$, which can then produce a performative proposition $\mathsf{Perform}_{\text{A}}([\text{stit}]\,\mathtt{Sat}(\mathsf{N}))$, etc. All that enables natural formalisation of informal normative inferences, like the examples in Section 2.3, into formal propositional logical inferences in a MS-Deon-based deductive system (see Section 4).

Expressing 'To-Be' norms. MS-Deon can express uniformly both agentive (indicated by the optional index A) and non-agentive 'To-Be' norms and normative propositions from the respective 'To-Do' norms and propositions:

	Norms	Norm-propositions
$\mathsf{Ought}^{be}_{(\text{A})}\varphi :=$	$\mathsf{Ought}^{do}_{(\text{A})}[\text{stit}]\varphi$	$\mathtt{InForce}(\mathsf{Ought}^{do}_{(\text{A})}[\text{stit}]\varphi)$
$\mathsf{Perm}^{be}_{(\text{A})}\varphi :=$	$\mathsf{Perm}^{do}_{(\text{A})}[\text{stit}]\varphi$	$\mathtt{InForce}(\mathsf{Perm}^{do}_{(\text{A})}[\text{stit}]\varphi)$
$\mathsf{Proh}^{be}_{(\text{A})}\varphi :=$	$\mathsf{Proh}^{do}_{(\text{A})}[\text{stit}]\varphi$	$\mathtt{InForce}(\mathsf{Proh}^{do}_{(\text{A})}[\text{stit}]\varphi)$

These apply likewise to produce normative propositions like $\mathtt{Sat}(\mathsf{Ought}^{be}\varphi)$.

Expressing individual conditional norms Individual conditional norms, e.g. of the type "*if agent* A *satisfies property* ϕ *then* A *ought to do* α" can be expressed as the agentive conditional norm $\mathsf{Ought}^{do}(\phi(\mathsf{a}) \wedge \psi_{\mathrm{A}}(\mathsf{a}), \alpha)$ if ψ_{A} is a characteristic property that is satisfied by A and by no other agent. Otherwise, expressing that is still possible, but in a somewhat cumbersome roundabout way: first, take the proposition $\psi = \phi(\mathrm{A}) \rightarrow \mathtt{InForce}(\mathsf{Ought}^{do}_{\mathrm{A}}\alpha)$ which can then be turned into a norm by using the construct $\mathsf{Ought}^{be}(\psi)$.

Adding imperatives. MS-Deon can be extended with a sort for *imperatives*, which can be both individual (referring to a specific agent, indicated below by the optional index (A)) and common, involving for instance constructs like:

$$[\mathtt{stit}\ (\mathrm{A})]!\varphi \mid \mathsf{Do}_{(\mathrm{A})}\alpha \mid \mathsf{Don't}_{(\mathrm{A})}\alpha \mid \mathsf{MayDo}_{(\mathrm{A})}\alpha$$

The construct $[\mathtt{stit}\ ]!\varphi$ intuitively means to express the common imperative 'See to it that ϕ!', whereas $[\mathtt{stit}\ \mathrm{A}]!\varphi$ means the same, but addressing only the agent A; the rest are self-explanatory. The idea of adding imperatives is that they can be applied by authorities or agents for producing new norms, i.e. bringing about obligation, permission, or prohibition requirements in force, thus enabling the expression of *norm creation.* This idea will be explored further in a follow-up work.

Expressing some deontic principles and relationships. MS-Deon enables expressing various *to-do* norms in a uniform way, by translating them to *seeing-to-it* norms, whenever appropriate, following the scheme:

'Agent A *ought-to-do X*' $\Rightarrow$
'Agent A *ought to see-to-it-that (or, ought to bring-it-about-that) X is done.*

Respectively,
'*X ought-to-be-done*' $\Rightarrow$
'It *ought-to-be-seen-to-it-that (ought-to-be-brought-about-that) X is done.*

On the other hand, MS-Deon also enables expressing the agentive *stit* action by the non-agentive one:

$$[\mathrm{A}\ \mathtt{stit}]\varphi := \mathsf{Perform}_{\mathrm{A}}([\mathrm{stit}]\varphi)$$

Further, the agentive *to-do* norms can be reduced to non-agentive *to-be* ones by using the well-known *Meinong-Chisholm Reduction* principle:
'*Agent* A *ought to see to it that p holds iff*
it ought to be that agent A *sees to it that p holds*'.
or,
'*Agent* A *is obliged to bring it about that p holds iff*
it is obligatory that agent A *brings it about that p holds*'.

The Meinong-Chisholm reduction principle is simply formalised in MS-Deon:

$$\mathsf{Ought}^{do}_{\mathrm{A}}[\mathrm{stit}]\varphi \equiv \mathsf{Ought}^{be}[\mathrm{A}\ \mathtt{stit}]\varphi$$

which, when translated back becomes:

$$\mathsf{Ought}^{do}_{\mathrm{A}}[\mathrm{stit}]\varphi \equiv \mathsf{Ought}^{do}[\mathrm{stit}]\mathsf{Perform}_{\mathrm{A}}([\mathrm{stit}]\varphi)$$

(Note the two different uses of 'stit' above.)

On the other hand, intuitively, the following equivalence should hold:

$$\mathsf{Ought}^{do}_{\mathrm{A}}\alpha \equiv \mathsf{Ought}^{be}[\mathrm{A\ stit}]\mathsf{Perform}_{\mathrm{A}}(\alpha).$$

This and other such intuitive validities impose natural requirements on the formal semantics.

4 Towards a system of deduction for MS-Deon

The main purpose of a logic-based framework is to provide a platform for logical deduction. This is especially the case for MS-Deon. Indeed, the full language seems too rich and generic to build a complete, yet feasibly complex logical system for it. So, in reality only suitable fragments of MS-Deon would be used for specific purposes.

The many-sortedness of MS-Deon suggests two different approaches to building systems of deduction for it:

(i) *many-sorted deduction*, involving inter-sort inference rules and allowing for the derivation of logical consequences not only in terms of propositions, but also in terms of norms. That, inter alia, resolves the dilemma of 'deontic logic of norms' vs 'deontic logic of normative propositions', by putting both together.

(ii) *'flat' single-sort deduction*, where reasoning about actions and norms is transformed to reasoning about performative and normative propositions.

The 'flat', proposition-based approach is the more traditional approach and can be naturally based on any standard logical deductive system e.g. a suitably enriched system of natural deduction, sequent calculus, or even a Hilbert-style system for axiomatic deduction [10], also involving intermediate steps of inter-sort transformations. A system of many-sorted deduction would be necessarily rule-based [11] and closer to a natural deduction style, which I find more practically useful, though also perhaps more challenging. Thus, both approaches have pros and cons, but, eventually, due to the inter-sort reductions, both should yield the same deductive power.

In either case, building a practically useful system of deduction which is sound both with respect to the prevailing common sense and (when the common

[10] As pointed out by a reviewer, such Hilbert-style system would be of limited practical use. Still, such system can be used as a starting point for building a more efficiently structured deductive systems, involving mechanisms for goal-oriented proof search. Resolution based proof systems are case in point.

[11] The rules are needed to avoid the clash of sorts that would occur if these were replaced by implications within formulae.

sense is inconclusive) with respect to the formal semantics provided further, and is also sufficiently rich to capture non-trivial normative inferences, is a big and long-term project which goes beyond the limitations of this paper. Still, here are some initial construction steps for a rule-based system of deduction for MS-Deon, with some relevant references:

- To begin with, such system should contain as a core a system of deduction (e.g., natural deduction or sequent calculus) [12] for the underlying classical logical reasoning. That could be a version or a fragment of some complete for first-order natural deduction, cf. e.g. [29,37], or [12].
- In addition, it should contain subsystems of rules capturing the logical properties of each of the other two main sorts, viz. actions and norms. For the sub-system of reasoning about actions built compositionally, in PDL style, possibly suitable deductive systems to build on are [19], [18], [9] and particularly [11], which explicitly incorporates the many-sorted approach. As for logics of norms, I am aware of few works on structured, rule-based deduction systems, starting with the pioneering [10], the non-technical but conceptually very relevant to the present work [13], [15] and the more recent [30], [8].
- The most challenging task is to develop a sufficiently rich system of *inter-sort rules* that enable truly many-sorted reasoning. Some of the earlier mentioned works enable structured multi-sorted deductive reasoning, but very few works that I am aware of, incl. [7] and [11], focus on that issue, and none of them on many-sorted *normative* reasoning.

Here are a few samples of many-sorted rules needed for formalising deductive reasoning in MS-Deon:

- If any agent a satisfying the condition φ ought to perform the action α and the agent $\textsc{a}$ satisfies the condition φ, then $\textsc{a}$ ought to perform the action α:

$$\frac{\mathtt{InForce}(\mathsf{Ought}^{do}(\varphi(\mathsf{a}), \alpha)), \quad \varphi(\textsc{a})}{\mathtt{InForce}(\mathsf{Ought}^{do}_{\textsc{a}}\alpha)}$$

  Hereafter, to keep the rules simpler, I will omit the construct $\mathtt{InForce}(\cdot)$ but will just write the norm N itself as a premise or conclusion of a rule, meaning that it represents the normative proposition $\mathtt{InForce}(\mathsf{N})$. An advantage of the many-sorted framework is that it allows such freedom of expression without formally producing the sort mismatch that would occur if N itself, rather than $\mathtt{InForce}(\mathsf{N})$, is used in a formula in a flat language of propositions.
- If any agent a is permitted to perform the action α and $\textsc{a}$ performs α then

[12] I do not mention here tableaux-based systems, because they are based on proofs by contradiction, which only applies when one already knows what conclusion one wants to prove, whereas practical normative reasoning is often open-ended. However, I should at least mention tableaux-based systems for deontic and other related logics by Rönnedal, cf. [31].

A's performing that action is legal:

$$\frac{\mathsf{Perm}^{do}_{\text{A}}\alpha,\ \mathsf{Perform}_{\text{A}}(\alpha)}{\mathtt{Legal}_{\text{A}}(\alpha)}$$

- If an agent A is prohibited from (respectively, obliged to) performing the action α and A performs (respectively, does not perform) α then A is violating that prohibition (respectively, obligation) norm:

$$\frac{\mathsf{Proh}^{do}_{(\mathsf{a})}\alpha,\ \mathsf{Perform}_{\text{A}}(\alpha)}{\neg\mathtt{Sat}_{\text{A}}(\mathsf{Proh}^{do}_{(\mathsf{a})}\alpha)},\qquad \frac{\mathsf{Ought}^{do}_{\mathsf{a}}\alpha,\ \neg\mathsf{Perform}_{\mathsf{a}}(\alpha)}{\neg\mathtt{Sat}_{\text{A}}(\mathsf{Ought}^{do}_{(\mathsf{a})}\alpha)}$$

- If any agent a is prohibited from seeng to it that φ, and if φ obtains for sure after a performs the action α, then a is prohibited from performing α:

$$\frac{\mathsf{Proh}^{do}_{(\mathsf{a})}[\text{stit}]\varphi,\ [\alpha]_{\mathsf{a}}\varphi}{\mathsf{Proh}^{do}_{\mathsf{a}}\alpha}$$

A stronger rule is obtained by replacing the premise $[\alpha]_{\mathsf{a}}\varphi$ with $\langle\alpha\rangle_{\mathsf{a}}\varphi$.

To illustrate how the MS-Deon framework can be used to formalise commonsense normative reasoning, consider the first example from Section 2.3. Let

- β be the action "*buying beer from the liquor store.*"
- $o(\mathsf{a})$ be the agent-parameterised atomic proposition "*a is over 20 years old.*"

Here is a formalised many-sorted derivation of that example, skipping a few trivial steps. Note the use of the (ad hoc introduced) inter-sort inference rules.

(i) $\mathsf{Perform}_{\mathsf{Pippi}}(\beta)$ (action performative proposition)

(ii) $\mathsf{Perm}^{do}(o(\mathsf{a}),\beta)$ (norm (permission))

(iii) $o(\mathsf{Pippi})$ ((essentially given) fact)

(iv) *Inter-sort inference rule*:

$$\frac{o(\text{A}),\ \mathsf{Perm}^{do}(o(\mathsf{a}),\beta)}{\mathsf{Perm}^{do}_{\text{A}}\beta}$$

(v) $\mathsf{Perm}^{do}_{\mathsf{Pippi}}\beta$ (individual norm, derived from i-iv)

(vi) *Inter-sort inference rule*:

$$\frac{\mathsf{Perm}^{do}_{\text{A}}\beta}{\mathtt{Legal}_{\text{A}}(\beta)}$$

(vii) $\mathtt{Legal}_{\mathsf{Pippi}}(\beta)$ (normative proposition, derived from v-vi)

The inference above can be transformed to a flat propositional inference, by using the $\mathtt{InForce}(\mathsf{N})$ construct to convert norms to normative propositions. Then, the inter-sort inference rules can be transformed to axiom schemes.

I leave the systematic development of a suitable full-fledged system of deduction for MS-Deon to future follow-up work.

Lastly, I just note here that many well-known deontic paradoxical inferences, such as Prior's paradoxes of derived obligations, are naturally blocked in a many-sorted inference system for MS-Deon because of involving disallowed inter-sort inferences; again, this is to be discussed in a future work.

5 Semantics for MS-Deon

Designing adequate formal semantics for such rich language as MS-Deon is not less challenging task than designing an adequate system of deduction for it. That semantics is still partly under construction, as some clauses depend on resolving quite non-trivial questions beyond the scope of this paper, e.g. of how norms extend from atomic over to composite actions. I will first present some basic principles behind the semantics, then will define the formal models, and then will give the semantic clauses for a large fragment of the language, leaving some clauses subject to additionally specified conditions resolving the issues mentioned above.

5.1 Semantics intuitively: main features

The semantics proposed here is influenced by several related works mentioned earlier and blends features from several semantics of well-known logics, incl. PDL, STIT, and the Coalition Logic CL, with some key new ideas:

- Models consist of states, describing snapshots of the world. Each state has:
  – a *propositional label*, being the set of atomic propositions true there;
  – a *normative label*, describing the atomic actions that are obligatory, permitted, or forbidden (commonly, or for a given agent) to perform from that state.
- Agents act concurrently from states of the model, each choosing independently to perform *a set of atomic actions* available to her at that state. (Such multiple actions are often referred to by norms, e.g. 'do not drink while driving'.) The result is an *action choice set profile*. Then, every agent performs independently all actions in her choice set (and only them).
- Transitions between states occur in a discrete manner and are determined by all agents performing 'simultaneously' their choices of actions. Sequences of states and transitions between them form *histories*.
- The composite actions are computed over histories, like in PDL.
- Norms are evaluated for validity (being *in-force*) at states, and for *agents' compliance* at histories and current transitions, in terms of the choice profile labels, inductively on their structure.
- Factual propositions are evaluated at states. Normative and performative propositions are evaluated at histories and current transitions.

5.2 Multi-agent deontic models

Given fixed (and usually assumed finite) sets of atomic propositions PROP, atomic actions ACT, and agents Agt, a **multi-agent deontic (MAD) model** over these is a structure

$$\mathcal{M} = \langle St, \mathsf{act}, \pi, \delta, \Delta, \tau \rangle, \quad \text{where:}$$

- St is a set of *states*.
- $\mathsf{act} : St \times \mathsf{Agt} \to \mathcal{P}(\mathsf{ACT})$ – mapping assigning to every state s and agent A a set $\mathsf{act}(s, \mathrm{A})$ of *actions available to* A *at* s.
- $\pi : St \to \mathcal{P}(\mathsf{PROP})$ is a *state description function*, assigning to each state s its *propositional label* $\pi(s)$.
- $\delta : St \times \mathsf{Agt} \to \mathcal{P}(\mathsf{ACT}) \times \mathcal{P}(\mathsf{ACT}) \times \mathcal{P}(\mathsf{ACT})$ is a *normative function*, such that $\delta(s, \mathrm{A}) = (\delta_o(s, \mathrm{A}), \delta_p(s, \mathrm{A}), \delta_f(s, \mathrm{A}))$ is the *normative label* of s, consisting of the sets of obligatory, permitted and forbidden actions for each agent A at state s. These satisfy the following natural constraints [13] :
  $\delta_o(s, \mathrm{A}) \subseteq \delta_p(s, \mathrm{A})$,
  $\delta_p(s, \mathrm{A}) \cap \delta_f(s, \mathrm{A}) = \emptyset$, and
  $\delta_p(s, \mathrm{A}) \cup \delta_f(s, \mathrm{A}) \subseteq \mathsf{act}(s, \mathrm{A})$.
- a *general normative function* $\Delta : St \to \mathcal{P}(\mathsf{ACT}) \times \mathcal{P}(\mathsf{ACT}) \times \mathcal{P}(\mathsf{ACT})$ is defined likewise, to specify the *general normative label* of s, applying to all agents [14] .
- $\tau : St \times \mathcal{P}(\mathsf{ACT})^{\mathsf{Agt}} \to St$ is the *transition function*, which for every $s \in St$ and a *choice of actions profile* σ determines the *successor state* $\tau(s, \sigma)$ of s, where a *choice of actions profile* at a state s is a mapping $\sigma : \mathsf{Agt} \to \mathcal{P}(\mathsf{ACT})$ such that $\sigma(\mathrm{A}) \subseteq \mathsf{act}(s, \mathrm{A})$ for each $\mathrm{A} \in \mathsf{Agt}$, representing the selection of available actions that the agent chooses to perform at the current state. Thus, the mapping σ is not part of the description of the model, but is a component of the context of evaluation of norms and propositions.

A *history* [15] in $\mathcal{M}$ is a finite sequence of states and the transitions between them. The last state of the history h will be denoted by $l(h)$, and $h \circ s$ will denote the history h extended with the state s.

5.3 Truth, norm validity, norm compliance, performance values

Given a MAD model $\mathcal{M} = \langle St, \mathsf{act}, \pi, \delta, \Delta, \tau \rangle$ we define *truth of propositions* $\models$, *validity of norms* $\models\!\!\!-$, and then *compliance with norms*, with respect to pairs

[13] The semantics presented here only handles norms that are applied in a particular situation (state), but not norms that are *not* applied because of being in conflict with higher priority norms applied at that state. Thus, the issue of conflicting norms and mechanisms for their resolution comes up here, which will not be addressed in the present work.

[14] Of course, Δ can be subsumed in all individual normative labels, but that would lead to an unnecessary repetition of all common norms for all existing and newly appearing agents.

[15] NB: this is distinct from the notion of 'history' in STIT models, where it is a primitive abstract entity, representing a potentially infinite possible course of events.

(history, choice profile) [16] in $\mathcal{M}$ by a mutual induction as follows (omitting the standard boolean cases, and given here only for atomic actions):

- $\mathcal{M}, h, \sigma \models p$ iff $p \in \pi(l(h))$.
  (Here $l(h)$ is the current state of the evolution of the system with history h.)
- $\mathcal{M}, h, \sigma \models \mathsf{Ought}^{do}\alpha$ iff $\alpha \in \Delta_o(l(h), \mathrm{A})$; likewise for $\mathsf{Perm}^{do}\alpha$ and $\mathsf{Proh}^{do}\alpha$.
  $\mathcal{M}, h, \sigma \models \mathsf{Ought}^{do}_{\mathrm{A}}\alpha$ iff $\alpha \in \delta_o(l(h), \mathrm{A})$; likewise for $\mathsf{Perm}^{do}_{\mathrm{A}}\alpha$ and $\mathsf{Proh}^{do}_{\mathrm{A}}\alpha$.
  We read $\mathcal{M}, h, \sigma \models \mathsf{N}$ as "The norm N is in force at $\mathcal{M}, h, \sigma$".
  Thus, the normative labels at the current state determine the norms regarding atomic actions that are in force at that state [17].
- $\mathcal{M}, h, \sigma \models \mathsf{Ought}^{do}(\varphi(\mathsf{a}), \alpha)$ iff $\alpha \in \delta_o(l(h), \mathrm{A})$ holds for all agents A such that $\mathcal{M}, h, \sigma \models \varphi(\mathrm{A})$ (assuming that each $\varphi(\mathrm{A})$ is already evaluated).
  Likewise for $\mathsf{Perm}^{do}(\varphi(\mathsf{a}), \alpha)$ and $\mathsf{Proh}^{do}(\varphi(\mathsf{a}), \alpha)$.
- $\models$ is extended to all norms inductively on the construction of norms, following additionally specified semantics of the norm-building operations.
- Now, we define $\mathcal{M}, h, \sigma \models \mathtt{InForce}(\mathsf{N})$ iff $\mathcal{M}, h, \sigma \models \mathsf{N}$,
- $\mathcal{M}, h, \sigma \models \mathtt{Sat}_{\mathrm{A}}(\mathsf{Ought}^{do}_{\mathrm{A}}\alpha)$ iff $\alpha \in \sigma(\mathrm{A})$ [18];
  $\mathcal{M}, h, \sigma \models \mathtt{Sat}_{\mathrm{A}}(\mathsf{Perm}^{do}_{\mathrm{A}}\alpha)$ iff $\alpha \in \delta_p(s, \mathrm{A})$;
  $\mathcal{M}, h, \sigma \models \mathtt{Sat}_{\mathrm{A}}(\mathsf{Proh}^{do}_{\mathrm{A}}\alpha)$ iff $\alpha \notin \sigma(\mathrm{A})$;
  Analogously for $\mathtt{Sat}(\mathsf{Ought}^{do}\alpha)$, $\mathtt{Sat}(\mathsf{Perm}^{do}\alpha)$, $\mathtt{Sat}(\mathsf{Proh}^{do}\alpha)$.
  The clauses for compliance with composite actions, providing semantics of $\mathtt{Legal}_{\mathrm{A}}(\alpha)$, are to be given inductively on the definition of actions, according to externally specified conditions. These are generally non-trivial and subject to ongoing research and debate, so they will not be provided here.
- $\mathcal{M}, h, \sigma \models \mathsf{Perform}_{\mathrm{A}}(\alpha_{at})$ iff $\alpha_{at} \in \sigma(\mathrm{A})$;
- $\mathcal{M}, h, \sigma \models \mathsf{Perform}_{\mathrm{A}}([\text{stit}]\phi)$ iff $\mathcal{M}, h, \sigma' \models \phi$ for every σ' with $\sigma'(\mathrm{A}) = \sigma(\mathrm{A})$
- The clauses for performance of composite actions are given inductively on their structure, according to their operational semantics, as in PDL.
- $\mathcal{M}, h, \sigma \models [\alpha]_{\mathrm{A}}(\phi)$ iff $\mathcal{M}, h \circ \tau(l(h), \sigma'), \sigma'' \models \phi$ for every σ' such that $\alpha \in \sigma'(\mathrm{A})$ and every choice of actions profile σ'' at $\tau(l(h), \sigma')$.
  Likewise for $\mathcal{M}, h, \sigma \models [\alpha](\phi)$.

[16] The truth of some propositions, e.g. the factual ones, would only depend on the current state, but others – typically normative and performative – also depend on the history and the choice profile.

[17] An essential point, raised by Karl Nygren: there is a difference between "existence" of norms and norms being "in force". The former typically refers to laws or other agentless and timeless norms, whereas the latter apply locally, at "states" possibly involving both a time instant and a location, i.e. apply "here and now". Ideally, both types of norms should be included and treated on a par in the formal framework. However, to keep it simpler, here I will assume that globally existing norms are included in all normative labels of states.

[18] This definition makes the provision that one can comply with a norm even if it is not in force (by being in the current normative label). Thus, we can keep the judgment of compliance/non-compliance separate from the validity of norms. That may be useful e.g. for counterfactual normative reasoning.

- The cases of agent-parameterised propositions will not be treated in full generality here, to avoid having to deal with assignments over agents. So far, we will only be interested in formulae where all agent-parameterised propositions are in the scope of conditional agentive norms, where their use is reduced to checking their truth for each agent in Agt.

This completes the semantics, modulo the mentioned externally specified mechanisms, e.g., for the extension of deontic operators to composite actions.

6 Some related work

In addition to the earlier cited works by von Wright, here is a selective and inevitably incomplete list of earlier publications, some of which have inspired, and others – just anticipated, some ideas in the present work.

- In [1] C. Alchourrón proposed a "normative logic", viz. a logic of normative propositions (rather than a deontic logic of norms). These are propositions stating that an agent has 'issued a norm'. The logic has two sorts of formulae, building on top of a modal deontic logic in early von Wright style, by applying a 'norm issuing' operator N to deontic formulae. A couple of years later, Alchourrón and Bulygin proposed in their 1971 book [2] another two sorted deontic language, involving a 'universe of actions' and 'universe of properties', with deontic operators applying to the former, to build normative systems, all in a semi-formal style.
- Castañeda proposes in [4] a 'calculus containing proposition-practition distinction', where he distinguishes ought-to-do norms applying to actions ('practitions'), from those applying to action propositions, but without providing formal semantics or a system of deduction for these.
- Several researchers have proposed (with different motivations) and developed essentially two sorted deontic logics of actions, by analogy with the (propositional) dynamic logic of programs PDL. In particular:
  - · K. Segerberg [32,33,34] developed a 'dynamic deontic logic' of actions, involving formulae and event (action) terms, evaluated over histories of events. *Norms* are defined only in the semantics, as functions N such that for any history h, $N(h)$ selects a set of extensions of h considered normal (legal) according to that norm. Still, Segerberg's system is closer to a logic of normative propositions, than to a logic of norms.
  - · J.-J. Meyer proposed in [27] a different approach to two-sorted deontic logic as a variant of dynamic logic. He revisited 'Segerberg style' of dynamic deontic logic in [28].
  - · P. Kulicki and R. Trypuz have developed in a series of papers [22,35,36,23] 'deontic action logics', based on algebraic approach to actions. These are formally logics of normative propositions about actions, not logics of norms.
  - · Other two-sorted deontic action logics were proposed by J. Broersen [3] and P. Castro and T. Maibaum [5].
- J. Hage has proposed and developed rule-based methodology and systems

for normative and legal reasonings in a series of non-technical but spiritually close to the present proposal works [19], including [14] (summarized in [13]), [15], and the more recent [16], amongst others.

- Motivated by the study of conditional norms, D. Makinson and L. van der Torre, proposed the "*Input-Output Logic*" framework [24,25,26]. Notably, they develop an inference mechanism not based on transfer of truth-values.
- M. Knobbout, M. Dastani and J.-J. Meyer introduce in [21] a deontic logic for distinct types of norms, viz. state-based and action-based, with semantics based on formally defined normative systems.

7 Concluding remarks

This paper stemmed from my, growing over the years, dissatisfaction with how modal logic – in the guise of 'standard deontic logic' – handles normative reasoning. I have argued here that a more elaborated formal framework for deontic logic is needed for adequately formalising (especially, multi-agent) normative reasoning. Then I have proposed such a many-sorted [20] formal framework, have outlined its language and formal semantics, and indicated briefly how it could be used for formalising normative reasoning. This work is still in progress and some features are still under construction, in particular the formal semantics for the full language and the development of adequate deductive systems for it.

I emphasise again that the proposed framework is generic and includes many syntactic features and constructs on actions, norms and propositions, only some of which would be applicable to any concrete instantiation of the framework. On the other hand, I also admit that a truly adequate logical formalisation of normative reasoning would require much more than what the proposed framework offers. It should also involve, *inter alia*:

- a full-fledged *first-order (or higher-order) language* for the domain of normative discourse, including constant, function, and predicate symbols, as well as quantification over individuals.
- an elaborated *multi-agent framework*, involving individual, as well as group and collective actions and respective deontic operators for individual, group, and collective obligations, permissions, and prohibitions, in their interaction; possibly, also explicit quantification over agents.
- *multi-agent knowledge*: individual, common and distributed, as well as *beliefs*. These are crucial for realistic and meaningful normative reasoning. Further, a more *refined theory of agency*, that relates the deontic attitudes with agents' knowledge, beliefs, and abilities to act is needed.
- *explicit temporality*, as norms exist and agents act over time, so judging norm compliance and violation *must* take temporality and timing into account.

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[19] I have only discovered these publications during the last days of work on the final version of this paper, hence my comments are brief and much more superficial than deserved.

[20] As noted by a reviewer, the virtues of using sorted/typed logical formalisms for knowledge representation and reasoning go well beyond normative reasoning and deontic logics.

All these are inherent aspects of adequate normative reasoning and should all be brought together, under one (enormous, but necessary) formal umbrella. That is a long-term (and multi-agent) endeavour, left to future ought-to-do's.

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