

Studies in Logic

Volume 61

Philosophical Applications of Modal Logic

Volume 50
Intuitionistic Set Theory
John L. Bell

Volume 51
Metalogical Contributions to the Nonmonotonic Theory of Abstract
Argumentation
Ringo Baumann

Volume 52
Inconsistency Robustness
Carl Hewitt and John Woods, eds.

Volume 53
Aristotle's Earlier Logic
John Woods

Volume 54
Proof Theory of N4-related Paraconsistent Logics
Norihiro Kamide and Heinrich Wansing

Volume 55
All about Proofs, Proofs for All
Bruno Woltzenlogel Paleo and David Delahaye, eds

Volume 56
Dualities for Structures of Applied Logics
Ewa Orłowska, Anna Maria Radzikowska and Ingrid Rewitzky

Volume 57
Proof-theoretic Semantics
Nissim Francez

Volume 58
Handbook of Mathematical Fuzzy Logic, Volume 3
Petr Cintula, Petr Hajek and Carles Noguera, eds.

Volume 59
The Psychology of Argument. Cognitive Approaches to Argumentation and
Persuasion
Fabio Paglieri, Laura Bonelli and Silvia Felletti, eds

Volume 60
Abstract Algebraic Logic. An Introductory Textbook
Josep Maria Font

Volume 61
Philosophical Applications of Modal Logic
Lloyd Humberstone

Studies in Logic Series Editor
Dov Gabbay

dov.gabbay@kcl.ac.uk

Philosophical Applications of Modal Logic

Lloyd Humberstone

© Individual author and College Publications 2015
All rights reserved.

ISBN 978-1-84890-196-4

College Publications
Scientific Director: Dov Gabbay
Managing Director: Jane Spurr
Department of Informatics
King's College London, Strand, London WC2R 2LS, UK

<http://www.collegepublications.co.uk>

Original cover design by Orchid Creative www.orchidcreative.co.uk
Printed by Lightning Source, Milton Keynes, UK

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form, or by any means, electronic, mechanical, photocopying, recording or otherwise without prior permission, in writing, from the publisher.

Contents

Preface	ii
1 Modal Logic	1
1.1 Introductory Overview	1
1.2 Non-Modal Background	5
1.3 Modal Logics: A Hierarchy of Classes	16
1.4 Refinements and Generalizations	30
2 Normal Modal Logics	33
2.1 Some Candidate Axioms	33
2.2 Models and Truth: Simplified Semantics	53
2.3 Models and Truth: Kripke Semantics	58
2.4 Canonical Models and Generated Submodels	61
2.5 From Models to Frames	71
2.6 The Rule of Disjunction	112
2.7 Revision Exercises	129
2.8 Supplement: Matsumoto's Embedding	142
2.9 Supplement: Matsumoto's Embedding (Concluded)	149
2.10 Semantical Postscript	152
A Quick History	152
Frames with Functions	157
An Application of Neighbourhood Semantics	171
3 Applications: Tense Logic	177
3.1 Axiomatizing the Basic Logic	177
3.2 Extensions of \mathbf{K}_t	183
3.3 Temporally Motivated Concerns: Density and Discreteness	194
4 Applications: Alethic, Nomic, Deontic	203
4.1 Introduction	203
4.2 Nomic Necessity I: Pargetter	217
4.3 Nomic Necessity II: Bacon	226
4.4 Deontic Logic: Main Themes	234
4.5 Deontic Logic: More Translations, More Issues	278
4.6 Logics Which Are Fully Modalized	290
4.7 "Nothing in Between": A Remark by A. N. Prior	304
4.8 The Fatal Disjunction: Danielsson on Ross's Paradox	324

5 Applications: Doxastic and Epistemic Logic	331
5.1 The Logical Omniscience Issue	331
5.2 Introspection Issues	370
5.3 Negative Introspection: Van der Hoek <i>et al.</i>	380
5.4 Negative Introspection: Halpern	385
5.5 Logics Between S4 and S5	402
5.6 Concept-Possession Problems	420
5.7 Another Use for B^{-1}	441
6 Coming to Stand in a Relation	449
6.1 Introduction	449
6.2 The Logic of Coming About	452
6.3 Relational Change and Coming About	456
6.4 Multiple Inchoativity	459
6.5 Cross-Predicability	460
6.6 Cross-Predicability and Multiple Inchoativity	467
6.7 Postscript: Completeness for the Logic of §6.2.	468
7 Appendix: Natural Deduction for S4 and S5	471
7.1 Non-Modal Rules	471
7.2 Natural Deduction Rules for \Box	475
7.3 Adding Rules for \Diamond	478
7.4 The Semantics of S5 and Some Rules for S4	481
7.5 Natural Deduction for Weaker Normal Modal Logics	486
References	494
Index	554

Philosophical Applications of Modal Logic

Preface

Modal logic, especially the range of what are called normal modal logics, taken collectively, has been applied to many areas of philosophical interest: to the representation of moral claims and principles (‘deontic logic’), to questions of knowledge and belief (‘epistemic logic’), and so on. It is to such philosophically motivated uses of modal logic – and specifically modal propositional logic (modal predicate being mentioned only in passing from time to time) – that the present work is devoted, though naturally the purely formal elaboration of the subject will require some attention first, before candidate applications can be reviewed. No fixed line is taken on the plausibility of the applications in question, but those wanting to make up their own minds will have many of the pertinent issues aired here in order to help them do so, together with plenty of references to the relevant literature. This should give the general idea of what is covered, though the following paragraph goes into more detail. In particular, while taking a great interest in the application of modal logic to various areas of philosophy, this book pays little attention to the philosophy of modal logic (or philosophy of logic more generally) and none to the philosophy (the metaphysics and epistemology, that is) of modality. The latter area has been treated extensively in many works, some of them (such as Forbes [319] and Lewis [726]) already in our list of references because of the logical issues they touch on, as well as some mentioned only here to redirect readers wanting such material to them: Jubien [633], and – for a more logic-oriented approach – Rini and Cresswell [959], as well as Stalnaker [1084], or Hale and Hoffmann [425] for a useful anthology.

Chapters 1 and 2 give the necessary technical background on the taxonomy of modal logics, their semantic treatment, especially for the normal modal logics, completeness and (what is called ‘global’) modal definability, the rule of disjunction and variations, etc., again, mostly as they arise for normal modal logics. Indeed somewhat more is provided than the bare technical prerequisites for subsequent chapters, when matters of logical interest arise naturally in the course of supplying such a background. And further, some technical matters are found in the later chapters when the application under discussion naturally calls for them or for this or that reason are most conveniently discussed there. For example, local (as opposed to global) modal definability is addressed in the ‘Applications’ chapter on tense logic – more specifically, in Section 3.2 – while a brief introduction to two-dimensional modal semantics can be found in the course of the discussion of deontic logic (after Example 4.4.45), in a chapter which also considers (Section 4.6) the issue of what have been called fully modalized logics, we well as in Section 5.7. Similarly, quite a bit of the discussion of non-normal modal logics is deferred to a point (p.360*ff.*) at which it becomes particularly relevant – in the discussion of deductive omniscience in epistemic logic (Section 5.1). Moral: in some cases the index will be of greater assistance than the contents pages for locating a topic of interest. It goes without saying, perhaps, that the topics chosen for discussion here reflect the author’s own interests, which are in broadly philosophical and conceptual matters, and those wanting material on, for example, questions of computational complexity, should look elsewhere. The same goes for those wanting an overview of the last thirty years’ advances in technical modal logic. Further prefatory material on notation, pre-requisites, etc., can be found at the end of Section 1.1; this includes suggestion for reading in the areas just described as not lying on the present agenda. In addition, I should mention that philosophically motivated departures from the standard language with one or more primitive one-place modal operators, such as the two-place conditional obligation operator often urged in deontic logic, are remarked on in passing but not given a detailed treatment – though plenty of pointers to the pertinent literature are supplied on such occasions.

This material started life as a lecture course in modal logic delivered to philosophy undergraduates at Princeton University in 2007. I am grateful to the Princeton Philosophy Department for inviting me to visit on that occasion and to those students who participated in the course for their feedback and their questions. I have since also had the benefit of assistance on this project from many people, including Rohan French, Allen Hazen, Wolfgang Lenzen, David Makinson and Evgeny Zolin, all of whom provided information or advice on specific topics. For their proof-reading assistance, I am greatly indebted to Rohan French and Sam Butchart.