Review of: Inconsistency Robustness (Carl Hewitt and John Woods assisted by Jane Spurr, eds.), College Publications, UK. 2015.

by

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This is an extraordinary book originating from two extraordinary conferences about a novel way of looking upon logical inconsistencies, Inconsistency Robustness 2011 and 2014, both held at Stanford University in California, USA, in the summers of 2011 and 2014. Instead of trying to avoid them (since in classical logic the whole thing explodes if there is an inconsistency, via the *ex falso quodlibet* rule), we are led to accept them (since in practice they appear everywhere), and reason with them in a non-classical way.

Inconsistency robust logic is an important conceptual advance in that requires that nothing "extra" can be inferred just from the presence of a contradiction. For example, suppose that there is a language with just two propositions, namely, *P* and *Q*. Furthermore, suppose that *P* and (**not** *P*) are axioms. Then, the only propositions that can be inferred in an inconsistency robust logic are (*P* **and** (**not** *P*)), ((**not** *P*) **and** (**not** *P*)), (*P* **or** (**not** *P*)), *etc.* In particular, (*P* **or** *Q*) cannot be inferred because otherwise *Q* could be erroneously inferred using (**not** *P*) by the rule of Disjunctive Syllogism. An example of a logic (called NanoIntuitionistic) which is *not* inconsistency robust has just one rule of inference, namely, classical proof by contradiction. NanoIntuitionistic is *not* inconsistency robust because (**not** *Q*), (**not** (**not** *Q*)), (**not** (**not** *P*)), *etc.* can be erroneously inferred from the contradictory axioms *P* and (**not** *P*). Note that *Q* cannot be inferred in NanoIntuitionistic (because there is no rule of double negation elimination). Consequently, NanoIntuitionistic is a paraconsistent logic (which was conceived by Stanisław Jaśkowski [Jaśkowski 1948] and then developed by many logicians to deal with inconsistencies in mathematical logic [Arruda 1989; Priest, and Routley 1989]) where a logic is by definition paraconsistent if and only if it is *not* the case that *every* proposition can be inferred from an inconsistency. In conclusion, a paraconsistent logic (*e.g.* NanoIntuitionistic) can allow erroneous inferences (*e.g.* (**not** *Q*)) from an inconsistency that are not allowed by inconsistency robustness. Of course, an inconsistency robust logic is also necessarily paraconsistent.

Previous approaches try to keep inconsistency as minimal as possible, while Hewitt's approach 'embraces' inconsistency as something that cannot be avoided and consequently must be dealt with. Hewitt's logic, called Direct Logic, has two variants: Classical Direct Logic (for classical mathematical theories thought to be consistent) and Inconsistency Robust Direct Logic (for possibly inconsistent theories) where the main difference is that the former has an *ex falso quodlibet* principle while the latter has not. Classical Direct Logic is used for the special case of mathematical theories known with high confidence to be consistent, *e.g*., plane geometry. Both variants of Direct Logic impose that propositions must be typed with the consequence that no (unlimited) "self-referential" sentences can be constructed such as the one used by Gödel to prove his incompleteness theorems. Direct Logic is based on argumentation, which may be viewed as a more computational approach than classical first-order logic.

As Hewitt says in his preface: *"The field of Inconsistency Robustness aims to provide practical, rigorous foundations for computer information systems having pervasively inconsistent information in a variety of fields e.g., computer science and engineering, health, management, law, etc."*

The approach defies Gödel's famous 2nd incompleteness theorem (traditionally deemed to be one of the greatest achievements in logic in the last century), which states that mathematics, if consistent, cannot prove its own consistency. In the Classical Direct Logic, mathematics is provably formally consistent! By formally consistent, it is meant that an inconsistency is not inferred. The proof is remarkably tiny consisting of only using proof by contradiction and soundness. In fact, it is so easy that one wonders why this was overlooked by so many great logicians in the past. The proof is also remarkable that it does not use knowledge about the content of mathematical theories (plane geometry, integers, *etc*.). The proof serves to formalize that consistency is built into the very architecture of classical mathematics. However, the proof of formal consistency does not prove *constructive* consistency, which is defined to be that the rules of Classical Direct Logic themselves do not derive a contradiction. Proof of constructive consistency requires a separate inductive proof using the axioms and rules of inference of Classical Direct Logic. The upshot is that, contra Gödel, there seems to be no inherent reason that mathematics cannot prove constructive consistency of Classical Direct Logic (which formalizes classical mathematical theories). However, such a proof is far beyond the current state of the art.

The book contains 14 chapters, organised into 3 parts: Mathematical Foundations, Software Foundations and Applications, an index and has 535 pages. The applications part contains chapters on inconsistency in legal reasoning, scientific ontology construction, linguistics, biology and chemistry, and the technological singularity. It contains an extensive preface by Carl Hewitt, in which the very idea of Inconsistency Robustness is motivated and explained intuitively, as well all papers are introduced and put into context.

I briefly go through the chapters. The first two chapters by Hewitt provide the foundations of DL with a focus on the foundations of mathematics and semantics. John Woods presents a well-wrought philosophical-historical perspective to Inconsistency Robustness, in which he BTW concludes that Direct Logic (in both variants) is still very much (admittedly very interesting) work in progress. Eric Kao discusses in a short article the role of principles like the law of excluded middle and proof by self-refutation in the 'explosive' character of IRDL. Part 2 deals with software foundations. In three articles Hewitt discusses the Actor Model of concurrent computation, the relation of DL with logic programming and ActorScript. At first sight there might seem to be no direct relation with inconsistency robustness. However, Actors are fundamental to the implementation of Direct Logic and its applications for the Internet of Things including issues of privacy and security. Then in Part 3, several application areas ranging from law to biology are discussed, as mentioned above.

References:

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